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Hidden symmetries of gauge potentials in supersymmetric gauge theories

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We discuss the transformation theory of spinor gauge potentials in extended supersymmetric Yang-Mills theories. The "hidden symmetry" transformations recently shown to carry a representation of an infinitely graded loop algebra are shown to generate symmetries of the spinor gauge potentials.

I. INTRODUCTION

The equations of motion of maximally extended supersymmetric Yang-Mills theories with Lagrangian having a bosonic sector

$$L=\mathrm{tr}\{-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{2}(D_\mu A_i)^2-\tfrac{1}{2}(D_\mu B_i)^2-\tfrac{1}{4}[A_i,A_j]^2 -\tfrac{1}{4}[B_i,B_j]^2-\tfrac{1}{2}[A_i,B_j]^2\}\ , \tag{1}$$

$\mu=0,1,2,3$; $i,j=1,2,3$ (in the notation of Ref. 1) admit a Lax-type linear system in superspace[2,3] which allows the construction of an infinity of conserved spinor currents[4] and implies that these equations possess an infinite-dimensional algebra of symmetry transformations isomorphic to the $\mathbb{Z}$ graded Lie algebra $G\otimes\mathbb{C}[\lambda,\lambda^{-1}]$ of G-valued Laurent polynomials in the arbitrary complex parameter λ (where G is the algebra of the gauge group).[4,5] In this paper we begin to study the significance of these formal results[6] which are reminiscent of completely integrable field-theory models in two dimensions and have therefore given rise to the hope that it is complete integrability which constrains the quantum dynamics at high momenta yielding a finite theory to all orders in perturbation theory. We shall elucidate the "hidden"-symmetry transformation properties of the spinor gauge potentials of the theory, showing that under these transformations the potentials are symmetric in the sense that they are left invariant modulo gauge transformations. We also discuss the relevance of these transformations for the superspace background-field method.

The field equations for the theory (1) take a very simple form in terms of a superfield spinor potential $(A^s_\alpha,A_{\dot\beta t})$, namely the following set of algebraic relations among the superfield curvatures:[2,7]

$$F^{st}_{\alpha\beta}+F^{st}_{\beta\alpha}=0=F_{\dot\alpha s,\dot\beta t}+F_{\dot\beta s,\dot\alpha t}\ ,\quad F^s_{\alpha,\dot\beta t}=0\ . \tag{2}$$

Here as usual[8] the gauge-covariant derivatives are given by

$$\nabla^s_\alpha=D^s_\alpha+A^s_\alpha\ ,\quad \overline{\nabla}_{\dot\alpha t}=\overline{D}_{\dot\alpha t}+A_{\dot\alpha t}\ ,\quad \nabla_{\alpha\dot\beta}=\partial_{\alpha\dot\beta}+A_{\alpha\dot\beta}\ ,$$

with a basis of tangent vector fields on superspace being given by

$$D^s_\alpha=\frac{\partial}{\partial\theta^\alpha_s}+i\overline{\theta}^{\dot\beta s}\partial_{\alpha\dot\beta}\ ,$$

$$\overline{D}_{\dot\beta t}=-\frac{\partial}{\partial\overline{\theta}^{\dot\beta t}}-i\theta^\alpha_t\partial_{\alpha\dot\beta}\ ,$$

$$\partial_{\alpha\dot\beta}=\frac{\partial}{\partial x^{\alpha\dot\beta}}\ .$$

The field strengths in (2) may be defined by the anticommutators

$$\{\nabla^s_\alpha,\nabla^t_\beta\}=F^{st}_{\alpha\beta}\ ,\quad \{\overline{\nabla}_{\dot\alpha s},\overline{\nabla}_{\dot\beta t}\}=F_{\dot\alpha s,\dot\beta t}\ ,$$

and

$$\{\nabla^s_\alpha,\nabla_{\dot\beta t}\}=F^s_{\alpha,\dot\beta t}-2i\delta^s_t\nabla_{\alpha\dot\beta}\ ,$$

which yields

$$F^s_{\alpha,\dot\beta t}=D^s_\alpha A_{\dot\beta t}+\overline{D}_{\dot\beta t}A^s_\alpha+\{A^s_\alpha,A_{\dot\beta t}\}+2iA_{\alpha\dot\beta}\delta^s_t\ .$$

The last constraint in (2) therefore determines the vector potential in terms of the spinor ones.

II. THE SYMMETRY TRANSFORMATIONS AND THEIR CLOSURE

We use the notation of Ref. 4 and we restrict our consideration to real superspace with the variables $y=(x^{\alpha\dot\beta},\theta^s_\alpha,\overline{\theta}^{\dot\beta t})$, $\alpha,\dot\alpha=1,2$; $s,t=1,\dots,N$, satisfying $y^\dagger=y$. The Lax-type system for these equations is given by

$$\begin{aligned}&(\nabla^s_1+\lambda\nabla^s_2)\chi=0\ ,\\ &(\nabla_{\dot 2t}+\lambda^{-2}\nabla_{\dot 1t})\chi=0\ ,\\ &(\nabla_{1\dot 2}+\lambda\nabla_{2\dot 2}+\lambda^{-1}\nabla_{2\dot 1}+\lambda^{-2}\nabla_{1\dot 1})\chi=0\ ,\end{aligned} \tag{3}$$

which is a special case of Witten's two-parameter set of equations

$$\sigma^\alpha\nabla^s_\alpha\chi=0=\mu^{\dot\alpha}\nabla^s_{\dot\alpha}\chi=\sigma^\alpha\mu^{\dot\beta}\nabla_{\alpha\dot\beta}\chi\ . \tag{4}$$

[Equations (3) and (4) are related by the identifications $\sigma^\alpha=(a,b)$, $\mu^\alpha=(a^2,b^2)$, $\lambda=b/a$.]

We take the superfield functional $\chi(\lambda)$ to have analyticity properties in the complex λ plane (cf. Ref. 9) such that either

$$\chi(\lambda)=\psi(\lambda)=\sum_{n=0}^{\infty}\lambda^n\psi^{(n)}$$

or (5)

$$\chi(\lambda)=\phi(\lambda)=\sum_{n=0}^{\infty}\lambda^{-n}\phi^{(n)} .$$

We denote the linear system symbolically as

$$\nabla^s(\lambda)\chi(\lambda)=[d^s(\lambda)+A^s(\lambda)]\chi(\lambda)=0 \tag{6}$$

so that the field equations (3) are equivalent to the statement that $A^s(\lambda)$ is a flat connection taking values in the loop algebra

$$A^s(\lambda)=\psi\, d^s\psi^{-1} , \tag{7}$$

or, equivalently,

$$A^s(\lambda)=\phi\, d^s\phi^{-1} . \tag{8}$$

It was shown in Ref. 4 that Eqs. (2) could be partially integrated by writing the spinor potentials in the pure-gauge form

$$\begin{aligned} &A_1^s=g^{-1}D_1^s g , \quad A_{\dot{1}t}=g^{-1}\bar{D}_{\dot{1}t}g , \\ &A_2^s=h^{-1}D_2^s h , \quad A_{\dot{2}t}=h^{-1}\bar{D}_{\dot{2}t}h , \end{aligned} \tag{9}$$

where the superfields g and h are given by $g^{-1}=\psi^{(0)}$, $h^{-1}=\phi^{(0)}$. The superfields g and h have the transformation properties

$$g\to e^{-S(\lambda)}ge^{\alpha} , \quad h\to e^{R(\lambda)}he^{\alpha} , \tag{10}$$

where $\alpha=\alpha(x,\theta,\bar{\theta})$ is the parameter of local gauge transformations taking values in the Lie algebra G, and

$$S(\lambda)=\sum_{n=0}^{\infty}\lambda^n S^{(n)}(x,\theta,\bar{\theta})$$

and

$$R(\lambda)=\sum_{n=0}^{\infty}\lambda^{-n}R^{(n)}(x,\theta,\bar{\theta})$$

are loop-algebra-valued infinitesimal parameters satisfying the Killing equations (cf. Ref. 10)

$$[\nabla^s(\lambda),S(\lambda)]=0=[\nabla^s(\lambda),R(\lambda)] , \tag{11}$$

whose solutions are the functional symmetry generators[11,12] with components that are the coefficients in the power-series expansions

$$S(\lambda)=\sum_{n=0}^{\infty}S^a_{(n)}\lambda^n T^a , \quad R(\lambda)=\sum_{n=0}^{\infty}R^a_{(n)}\lambda^{-n}T^a .$$

These components span infinite-dimensional vector spaces of symmetries. Equations (11) imply the field equations (2), and their general solution is given by S and R having the form of a similarity transformation of a constant matrix in the Lie algebra G,

$$S^a=\psi T^a\psi^{-1} , \quad R^a=\phi T^a\phi^{-1} , \tag{12}$$

where we have expanded in a basis of G. We shall refer to the α transformations in (10) as G transformations, and the S and R transformations as G^λ transformations. We note that under the G transformations the spinor potentials transform in the usual inhomogeneous manner:

$$A_A^s\to e^{-\alpha}A_A^s e^{\alpha}+e^{-\alpha}D_A^s e^{\alpha} ,$$

whereas under the G^λ transformations we have the parametric transformations:

$$\begin{aligned} &\delta(\lambda)A_2^s=h^{-1}D_1^s R(\lambda)h=[\nabla_2^s,h^{-1}R(\lambda)h] , \\ &\delta(\lambda)A_2^s=h^{-1}D_2^s R(\lambda)h=[\nabla_2^s,h^{-1}R(\lambda)h] , \\ &\delta(\lambda)A_{\dot{1}t}=-g^{-1}\bar{D}_{\dot{1}t}S(\lambda)g=-[\bar{\nabla}_{\dot{1}t},g^{-1}S(\lambda)g] , \\ &\delta(\lambda)A_{\dot{2}t}=h^{-1}\bar{D}_{\dot{2}t}R(\lambda)h=[\nabla_{\dot{2}t},h^{-1}R(\lambda)h] . \end{aligned} \tag{13}$$

We may therefore conclude that the spinor potential (9) is symmetric under the infinite set of G^λ transformations, since the variation of the potential $(A_\alpha^s,A_{\dot{\alpha}t})$ under each of these transformations, as given by the power-series expansion in λ of Eq. (13), may be undone by a gauge transformation leaving invariant the gauge condition:

$$\theta_s^\alpha A_\alpha^s-\bar{\theta}^{\dot{\alpha}t}A_{\dot{\alpha}t}=0 . \tag{14}$$

This "transverse" gauge condition was recently introduced in Ref. 13, where it was shown that it effectively eliminates all θ-dependent gauge transformations. Therefore, under each of the infinite set of G^λ transformations, the potential $(A_\alpha^s,A_{\dot{\alpha}t})$ is left invariant modulo x-dependent gauge transformations. Since S and R are nonlinear functionals of g and h [the functional dependence being determined by Eq. (11)], each coefficient in the power-series expansion of the G^λ transformation effects a nonlinearly realized G transformation on the potential, leaving Eq. (14) invariant.

Considering the condition for integrability to finite transformations[14] of these infinitesimal transformations, we obtain, for instance, by composing two particular G^λ transformations of A_1^t,

$$\begin{aligned} [\delta^b(\tau)\delta^a(\mu)-\delta^a(\mu)\delta^b(\tau)]A_1^t&=g^{-1}\{D_1^t(\delta^a(\mu)S^b(\tau)-\delta^b(\tau)S^a(\mu)+[S^a(\mu),S^b(\tau)])\}g \\ &=[\nabla_1^t,g^{-1}\{\delta^a(\mu)S^b(\tau)-\delta^b(\tau)S^a(\mu)+[S^a(\mu),S^b(\tau)]\}g] . \end{aligned} \tag{15}$$

If both these transformations are parametrized by $S(\lambda)$, the right-hand side of Eq. (15) is given by

$$\left[\nabla_1^t, C_{abc}g^{-1}\left[\frac{\mu S_c(\mu)-\tau S_c(\tau)}{\mu-\tau}\right]\right], \tag{16}$$

where the change in $S^b(\tau)$, $\delta^a(\mu)S^b(\tau)$, is the solution of

$$[\nabla^t(\tau),\delta^a(\mu)S^b(\tau)]+[\delta^a(\mu)A^t(\tau),S^b(\tau)]=0 ,$$

the condition for the invariance of (11); and it may be determined using the results of Ref. 4. Considering the term of order $\mu^m\tau^n$ in Eqs. (15) and (16), we find

$$\begin{aligned}[\delta^b_{(n)},\delta^a_{(m)}]A_1^t &= -[\nabla_1^t, C_{abc}g^{-1}S_c^{(n+m)}g] \\ &= C_{abc}\delta^c_{(n+m)}A_1^t, \text{ from (13)} ,\end{aligned} \tag{17}$$

demonstrating the closure of the algebra of these transformations (i.e., integrability). Similarly, a consideration of the composition of two particular R transformations of A_2^t yields, as the coefficient of $\mu^{-m}\tau^{-n}$, the expression

$$\begin{aligned}[\delta^b_{(-n)},\delta^a_{-(m)}]A_2^t &= -[\nabla_2^t, C_{abc}h^{-1}R_c^{(n+m)}h] \\ &= \delta^c_{-(n+m)}A_2^t .\end{aligned} \tag{18}$$

The generators of the infinitesimal transformation associated with the above transformations (cf. Ref. 15) may be defined by

$$L_a^n=\frac{d^n}{d\lambda^n}L_a(\lambda)\bigg|_{\lambda=0} , \quad L_n=\int dz\, \delta_a A_A \frac{\delta}{\delta A_A}$$
$$(dz=d^4x\, d^{2N}\theta\, d^{2N}\bar\theta) \tag{19}$$

a functional analog of the Lie derivative with respect to the functional symmetry generators $S^a_{(n)},R^a_{(n)}$ spanning the infinite-dimensional vector spaces of symmetries. The composition of two such generators yields

$$[L_a(\lambda),L_b(\mu)]=\int dz[\delta_a(\lambda),\delta_b(\mu)]A_A\frac{\delta}{\delta A_A} . \tag{20}$$

From (17) we clearly have the commutation relations

$$[L_a^m,L_b^n]=C_{abc}L_c^{m+n} \tag{21}$$

with $0\le m,n<\infty$; and from (18) we obtain the same commutation relations but with $-\infty<m,n\le 0$. We may therefore conclude that the spinor potential carries a representation of this subalgebra of the full loop algebra in the case when either $R(\lambda)=0$ and the two components of the potential, A_2^s, $A_{\dot 2s}$, are left invariant under the transformations, or when $S(\lambda)=0$ and the other two components, A_1^s, $A_{\dot 1s}$, remain unchanged. However, it is possible to consider a nontrivial action on the potential of all the G^λ transformations, those parametrized by $S(\lambda)$ as well as those parametrized by $R(\lambda)$. If we compose an S transformation of A_1^t with an R transformation, we obtain instead of Eq. (15)

$$\begin{aligned}[\delta_R^b(\tau)\delta_S^a(\mu)-\delta_S^a(\mu)\delta_R^b(\tau)]A_1^t &= \delta_R^b(\tau)\delta_S^a(\mu)A_1^t \\ &= -\delta_R^b(\tau)[\nabla_1^t,g^{-1}S^a(\mu)g] , \text{ from (13)} \\ &= -[\nabla_1^t,g^{-1}\delta_R^b(\tau)S^a(\mu)g] \\ &= -\left[\nabla_1^t,\frac{\tau\mu}{1-\tau\mu}g^{-1}[R_b(\tau),S_a(\mu)]g\right]-\left[\nabla_1^t,\frac{\tau\mu}{1-\tau\mu}C_{abc}S_c(\mu)\right] .\end{aligned} \tag{22}$$

The second term in (22) clearly corresponds to a G^λ transformation of A_1^t of the form given in (13). However, the first term corresponds to a gauge transformation more general than one induced by a G^λ transformation, albeit one which still leaves the gauge condition (14) invariant. The spinor potential therefore does carry a representation of the full loop algebra with commutation relations (21) with $m,n\in Z$, modulo such gauge transformations. To obtain the relations (21) explicitly, for all $m,n\in Z$, we have to modify the transformation rules (13) by allowing further transformations of the components of the potential corresponding to more general gauge transformations leaving (14) invariant.

From (10) it is clear that g and h can be used as bridges (cf. Ref. 16) to convert G-covariant objects into G^λ-covariant ones which are manifestly G invariant. This conversion was explicitly performed in Ref. 4 in order to display an infinity of conserved quantities and the closure of the algebra of symmetry transformations acting on the G^λ-covariant, manifestly G-invariant superfield $B=gh^{-1}$. Gauge transforming with respect to g^{-1}, the components of the potential become

$$\mathscr{A}_1^t=0 , \quad \mathscr{A}_2^t=BD_2^tB^{-1} , \quad \mathscr{A}_{\dot 1t}=0 , \quad A_{\dot 2t}=B\bar D_{\dot 2t}B^{-1} , \tag{23}$$

whereas rotating (9) by h^{-1}, we obtain

$$\tilde{\mathscr{A}}_1^t=B^{-1}D_1^tB , \quad \tilde{\mathscr{A}}_2^t=0 , \quad \tilde{\mathscr{A}}_{\dot 1t}=B^{-1}\bar D_{\dot 1t}B , \quad \tilde{\mathscr{A}}_{\dot 2t}=0 . \tag{24}$$

The superfield B is manifestly G invariant and transforms in the following covariant manner under G^λ transformations,

$$B\to e^{-S(\lambda)}Be^{-R(\lambda)} , \tag{25}$$

inducing the following infinitesimal parametric transformations on the potentials (23) and (24):

$$\delta(\lambda)\mathscr{A}^t_A=[\mathscr{D}^t_A,S(\lambda)]+[\mathscr{D}^t_A,BR(\lambda)B^{-1}] \quad (A=2,\dot{2}) , \tag{26}$$

$$\delta(\lambda)\tilde{\mathscr{A}}^t_A=-[\tilde{\mathscr{D}}^t_A,R(\lambda)]-[\tilde{\mathscr{D}}^t_A,B^{-1}S(\lambda)B] \quad (A=1,\dot{1}) , \tag{27}$$

where $\mathscr{D}^t_A=D^t_A+\mathscr{A}^t_A$, $\tilde{\mathscr{D}}^t_A=D^t_A+\tilde{\mathscr{A}}^t_A$. These transformations generate symmetries of the G^λ-covariant linear systems

$$\mathscr{D}^S(\lambda)X(\lambda)=0 \tag{28}$$

[where $X(\lambda)$ denotes either (case A) $g\psi(\lambda)\equiv\Psi(\lambda)$; $\Psi(\lambda=0)=\mathbb{1}$ or (case B) $g\phi(\lambda)\equiv\tilde{\Phi}(\lambda)$; $\tilde{\Phi}(\lambda=\infty)=B$] and

$$\tilde{\mathscr{D}}^s(\lambda)\tilde{X}(\lambda)=0 \tag{29}$$

[where $\tilde{X}(\lambda)$ denotes either (case B) $h\psi(\lambda)\equiv\tilde{\Psi}(\lambda)$; $\tilde{\Psi}(\lambda=0)=B^{-1}$, or (case A) $h\phi(\lambda)=\Phi(\lambda)$; $\Phi(\lambda=\infty)=\mathbb{1}$], by virtue of which the G^λ-covariant forms of the field equations (2),

$$\begin{aligned} &D^{(s}_1\mathscr{A}^{t)}_2=0=\bar{D}_{\dot{1}(s}\mathscr{A}_{\dot{2}t)} ,\\ &D^s_1\mathscr{A}_{\dot{2}t}+\delta^s_t 2ig\nabla_{1\dot{2}}g^{-1}=0 ,\\ &\bar{D}_{\dot{1}s}\mathscr{A}^t_2+\delta^t_s 2ig\nabla_{2\dot{1}}g^{-1}=0 ,\end{aligned} \tag{30a}$$

or, equivalently,

$$\begin{aligned} &D^{(s}_2\tilde{\mathscr{A}}^{t)}_1=0=\bar{D}_{\dot{2}(s}\tilde{\mathscr{A}}_{\dot{1}t)} ,\\ &\bar{D}_{\dot{2}s}\tilde{\mathscr{A}}^t_1+\delta^t_s 2ih\nabla_{1\dot{2}}h^{-1}=0 ,\\ &D^s_2\tilde{\mathscr{A}}_{\dot{1}t}+\delta^s_t 2ih\nabla_{2\dot{1}}h^{-1}=0 ,\end{aligned} \tag{30b}$$

are also left invariant under the infinitesimal parametric transformations (26) and (27).[4]

These results follow from the Killing equations for S and R (case A):

$$[\mathscr{D}^s(\lambda),S(\lambda)]=0=[\tilde{\mathscr{D}}^s(\lambda),R(\lambda)] ,$$

from which follow the relations for S and R given in Ref. 4, namely,

$$S=\Psi(\lambda)T\Psi(\lambda)^{-1} , \quad R=\Phi(\lambda)T\Phi(\lambda)^{-1} , \tag{31a}$$

or alternatively we have the relations (case B)

$$S=\tilde{\Psi}(\lambda)T\tilde{\Psi}(\lambda)^{-1} , \quad R=\tilde{\Phi}(\lambda)T\tilde{\Phi}(\lambda)^{-1} , \tag{31b}$$

which solve the equations

$$[\tilde{\mathscr{D}}^s(\lambda),S(\lambda)]=0=[\mathscr{D}^s(\lambda),R(\lambda)] .$$

A consideration of the integrability conditions $[\delta^b(\tau)\delta^a(\mu)-\delta^a(\mu)\delta^b(\tau)]\mathscr{A}^t_A$ and $[\delta^b(\tau)\delta^a(\mu)-\delta^a(\mu)\delta^b(\tau)]\tilde{\mathscr{A}}^t_A$ shows that the generators of the transformations (26) and (27)

$$N^n_a=\begin{cases} \int dz\, \delta^{(n)}_a\mathscr{A}_A , & n>0 \\ \int dz\, \delta^{(n)}_a\tilde{\mathscr{A}}_A\dfrac{\delta}{\delta\tilde{\mathscr{A}}_A} , & n<0 \end{cases} , \tag{32a}$$

for case A, and

$$\tilde{N}^n_a=\begin{cases} \int dz\, \delta^{(n)}_a\tilde{\mathscr{A}}_A\dfrac{\delta}{\delta\tilde{\mathscr{A}}_A} , & n>0 \\ \int dz\, \delta^{(n)}_a\mathscr{A}_A\dfrac{\delta}{\delta\mathscr{A}_A} , & n<0 \end{cases} , \tag{32b}$$

$$\tilde{N}^0_a=N^0_a ,$$

for case B, respectively, obey the commutation relations

$$[N^n_a,N^m_b]=C_{abc}N^{n+m}_c , \quad n,m\in Z \tag{33a}$$

and

$$[\tilde{N}^n_a,\tilde{N}^m_b]=C_{abc}\tilde{N}^{n+m}_c , \quad n,m\in Z . \tag{33b}$$

Considering the potentials (23) and (24) to be functionals of B, we have the functional Taylor expansion[17]

$$A_A[B+\delta B]=A_A[B]+\int dz\,\delta B\frac{\delta}{\delta B}A_A[B]+\cdots ,$$

which demonstrates that the generators (32a) are equivalent to the generators

$$M^n_a=\int dz\,\delta^{(n)}_a B\frac{\delta}{\delta B}$$

of the functional symmetry transformations in the solution space of (30), which were shown in Ref. 4 to close under the commutation relations (33a). Here we have explicitly exhibited the alternative set (case B) of functional symmetry transformations (25), with S and R given by (31b), generating the loop algebra with commutation relations (33b). Corresponding to these two sets of functional symmetry transformations, there exist two (completely equivalent) infinite sets of nonlocal conserved currents, which may be represented compactly by the following expression for the nth conservation law:

$$\{D^\alpha_s[\epsilon^\beta_\alpha\alpha^s_t\mathscr{D}^{(n)t}_\beta\chi_{(\beta)}(\lambda)]-D^{\dot{\alpha}t}[\epsilon^{\dot{\beta}}_{\dot{\alpha}}\alpha^s_t\mathscr{D}^{(n)}_{\dot{\beta}s}\chi_{(\dot{\beta})}(\lambda)]\}\,|_{\lambda=0}=0 , \tag{34}$$

where

$$\alpha^s_t=\begin{cases} 0 , & s=t \\ 1 , & s\neq t \end{cases} ;$$

and (case A)

$$\mathscr{D}^{(n)t}_A=\frac{1}{\lambda^n}\mathscr{D}^t_A , \quad \chi_{(A)}(\lambda)=\Psi(\lambda)=\sum_{n=0}^{\infty}\lambda^n\Psi^{(n)} , \quad A=2,\dot{2} ,$$

$$\mathscr{D}^{(n)t}_A=\lambda^n\tilde{\mathscr{D}}^t_A , \quad \chi_{(A)}(\lambda)=\Phi(\lambda)=\sum_{n=0}^{\infty}\lambda^{-n}\Phi^{(n)} , \quad A=1,\dot{1} ,$$

or alternatively (case B),

$$\mathscr{D}_A^{(n)t}=\lambda^n\mathscr{D}^t_A\ ,\ \ \chi_{(A)}(\lambda)=\widetilde{\Phi}(\lambda)=\sum_{n=0}^{\infty}\lambda^{-n}\widetilde{\Phi}^{(n)}\ ,\ \ A=2,\dot{2}\ ,$$

$$\mathscr{D}_A^{(n)t}=\frac{1}{\lambda^n}\mathscr{D}^t_A\ ,\ \ \chi_{(A)}(\lambda)=\widetilde{\Psi}(\lambda)=\sum_{n=0}^{\infty}\lambda^{n}\widetilde{\Psi}^{(n)}\ ,\ \ A=1,\dot{1}\ ,$$

The proof of Eq. (34) for case A is given in Ref. 4, and for case B follows that given there.

III. CONCLUDING REMARKS

Whether the structure described here, and in Refs. 2–6, has any more than merely a formal significance, in particular, whether this is the underlying classical structure constraining the quantum dynamics at high momenta (and yielding a finite theory to all orders in perturbation theory,[18,19] is still an open question. However, in two dimensions, the analogous conserved changes do constrain the dynamics, in fact determining the exact S matrix;[20] so the conjectured correspondence between integrability and finiteness is quite plausible. In the background-field formulation of the quantum theory, one may split the superfield g, for instance, into quantum (q) and classical background (b) factors: $g=qb$. Then, the components of the potential which are functionals of g split into background and quantum fields thus:

$$\begin{aligned}A^t_A=g^{-1}D^t_Ag&=(b^{-1}q^{-1})D_A(qb)\\&=b^{-1}D_Ab+b^{-1}(q^{-1}D_Aq)b\\&=A^{\rm cl}_A+a^q_A\ ,\end{aligned}$$

where $A^{\rm cl}_A=b^{-1}D_Ab$. Under G transformations ($b\to be^{\alpha}$) the background field transforms in the usual inhomogeneous manner

$$A^{\rm cl}_A\to e^{-\alpha}A^{\rm cl}_Ae^{\alpha}+e^{-\alpha}D_Ae^{\alpha}\ ,$$

whereas a^q_A transforms covariantly: $a^q_A\to e^{-\alpha}a^q_Ae^{\alpha}$. However, under G^{λ} transformations, e.g., $q\to e^{-s}q$, the background field remains unchanged $A^{\rm cl}_A\to A^{\rm cl}_A$, whereas the quantum field transforms inhomogeneously:

$$a^q_A=(b^{-1}e^sb)a^q_A(b^{-1}e^{-s}b)+b^{-1}(e^sD_Ae^{-s})b\ .$$

These transformation rules show that the parameter $S(\lambda)$ appears only in quantum transformations, whereas α appears only in the background transformations. This property is characteristic of the background-field formulation,[21] where one usually chooses for quantum gauge fields a background-covariant gauge-fixing term, thus obtaining a gauge-invariant effective action. The G^{λ} transformations here are reminiscent of the pregauge transformations used, for instance, in Ref. 19, to decouple ghosts from the theory. Whether these G^{λ} transformations can be similarly used, perhaps in some off-shell generalization of the maximal theory, remains to be seen.

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