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Matryoshka of special democratic forms. (English summary)

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Do not be put off by the odd title of this interesting article. In 7 dimensions, there is a special 3-form ω called the Cayley form with the property that the compact Lie group G_2 can be realised as the 7×7 matrices preserving ω. In fact, the Cayley form determines an inner product on $\mathbb{R}^7$ and an embedding $G_2\subset \mathrm{SO}(7)$. Like the volume form on a Euclidean vector space, the Cayley form has the property that there is an orthonormal basis in which its nonzero components are ±1 and this property is what the authors mean by 'special'. Like the volume form, the Cayley form also has the property that its nonzero components may be transitively permuted by a subgroup of $\mathrm{O}(7,\mathbb{Z})$, such a form being called 'democratic'. Examples of special democratic forms are (a) a $\mathrm{U}(3)$-invariant 2-form in 6 dimensions, (b) this G_2-invariant 3-form in 7 dimensions, and (c) a $\mathrm{Spin}(7)$-invariant 4-form in 8 dimensions.

A matryoshka is a set of Russian dolls designed to nest within one another and one has the impression that these particular examples are trying to nest as (a)⊂(b)⊂(c). The authors make this notion precise in terms of their symmetries. But then they find a bigger doll! There is a special democratic 6-form Ω in 10 dimensions with remarkable properties including the extension of this nesting to (a)⊂(b)⊂(c)⊂(d).

Reviewed by *Michael G. Eastwood*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.