Institut für Mathematik

Lehrstuhl für Wahrscheinlichkeitstheorie

Forschungsseminar im SoSe 2017

Theorie der stochastischen Prozesse

Freitags, 10:15-11:45, Golm, Haus 9 Raum 0.13

Fr. 21. April 2017 **Andrey Pilipenko** (Kyiv)

A functional limit theorem for excited random walks

We consider the limit behavior of an excited random walk (ERW), i.e. a random walk whose transition probabilities depend on the number of times the walk has visited the current state. We prove that an ERW being naturally scaled converges in distribution to an excited Brownian motion that satisfies an SDE, where the drift of the unknown process depends on its local time. Similar result was obtained by Raimond and Schapira, their proof was based on the Ray-Knight type theorems. We propose a new method based on a study of the Radon-Nikodym density of the ERW distribution with respect to the distribution of a symmetric random walk.

Mi. 26. April 2017 am Weierstraß-Institut, Berlin **Andrey Pilipenko** **(**Kyiv**)**

On a selection problem for small noise perturbation of ODE in multidimensional case

The identification problem of the limit of an ODE with non-Lipschitz drift perturbed by a zero-noise is considered in a multidimensional framework. This problem is a classical subject of stochastic analysis, however the multi- dimensional case was poorly investigated. We consider two cases in particular: (i) the drift coefficient has a jump discontinuity along an hyperplane and is Lipschitz continuous in the upper and lower half-spaces; (ii) the drift is equivalent to $a(\varphi)\, r^{\alpha}$ as r tends to 0, where (r,φ) are the polar coordinates, and $\alpha<1$.

Fr. 5. Mai 2017 **Giovanni Conforti** (Lille):

Approximating conditional distributions

There is a wealth of methods to bound the distance between probability laws. However, when we want to confront two laws conditioned to the outcome of some random experiment, several difficulties arises. On the one hand, conditioning introduces strong dependence between the different components of a model, and most of the methods work well in a regime of weak dependence. However, quite surprisingly, there are also many situations when very different laws have similar, or even identical conditional distributions. In this talk, we discuss the basic ideas of a general procedure to adapt Stein's method to bound the distance between conditional distributions and present two fundamental applications, where conditional laws arise naturally: the case of bridges and the filtering equation.
Joint work with A.Chiarini (Marseille) and A.Cipriani (Bath).

Fr. 12. Mai 2017 **Iurii Ganychenko** (Potsdam)

Strong and weak rates of approximation for integral functionals of Markov processes

We provide weak and strong rates of approximation of integral functionals of Markov processes by Riemann sums. Assumptions on the processes are formulated only in terms of their transition

probability densities and therefore are quite flexible. Namely, we pose a proper boundary condition on the derivative of the transition probability density of the respective Markov process with respect to the time variable. The class of processes under consideration includes diffusion processes, stable processes and models with Lévy(-type) noises.
We focus on integral functionals with non-regular kernels. As a particular important example of such a kernel, we consider an indicator function and the occupation time of a Markov process as a respective integral functional. We apply the results of weak and strong approximation rates of integral functionals to the estimates of the error of approximation of the price of an occupation time option.
Ganychenko, Iu., Knopova, V., Kulik A. (2015) Accuracy of discrete approximation for integral functionals of Markov processes, Modern Stochastics: Theory and Applications, Vol 2, No 4, 2015, 401 – 420.

Randolf Altmeyer (Berlin)

Estimating occupation time functionals

An occupation time functional is a Lebesgue integral of a function f(X_t) with X a stochastic process (X_t). Given the observations at discrete times (X_{k T/n})_k, we study the approximation of the occupation time by the Riemann-typeestimator.
When the process X is a continuous Itô semimartingale and the function f is smooth in the sense that its Fourier transform is sufficiently integrable, we obtain stable central limit theorems with 1/n as rate of convergence. This holds, in particular, for sufficiently "nice" X and weakly differentiable functions f. When X is a Brownian motion and f is weakly differentiable, we show that the rate of convergence and the asymptotic variance are optimal among all possible estimators, while the rate does not improve for more smooth f. The methods for proving the central limit theorems also yield generalized Itô formulas which are of independent interest and which we will discuss briefly.
We further present general conditions on X to obtain rates of convergence when f lies in some fractional L^2-Sobolev space of smoothness degree between 0 and 1, which explains rates obtained in the literature when f is a Hölder function or an indicator function of a bounded set. The conditions on X apply to many important classes of processes such as Markov processes, in particular diffusions and Lévy-processes, but also to non-Markovian processes, such as fractional Brownian motion. Again, for Brownian motion these rates are optimal.

Fr. 19. Mai 2017 **Olga Aryasova** (Kyiv)

On mutual behavior of solutions of an SDE with non-regular drift

We consider a stochastic differential equation with a Gaussian noise and a drift vector having jump discontinuities along a hyperplane or surface. We study the large time behavior of the distance between two solutions starting from different points.

Fr. 2. Juni 2017 **Jennifer Krüger** (Berlin)

Well-Posedness and stability of Stochastic Evolution Equations arising from Neuroscience

We study the existence and uniqueness of mild solutions to the deterministic and the stochastic neural field equation with Heaviside firing rate. Since standard well-posedness results do not apply in case of a discontinuous firing rate, we present a monotone Picard iteration scheme to show the existence of a maximal mild solution. Further, we illustrate that general uniqueness does not hold and therefore investigate uniqueness under suitable additional properties of the solutions. Here a novel criterion, the so-called absolute-continuity condition is introduced. Moreover, we observe regularisation by noise: With a suitable choice of spatially correlated additive noise uniqueness is restored without imposing any additional structural assumptions. In the second part of the talk we present a multiscale analysis of 1D stochastic bistable reaction-diffusion equations with additive noise. It is shown with explicit error estimates on appropriate

function spaces that up to lower order w.r.t. the noise amplitude, the solution can be decomposed into the orthogonal sum of a travelling wave moving with random speed and into Gaussian fluctuations. Our results extend corresponding results obtained for stochastic neural field equations to the present class of stochastic dynamics.

Fr. 23. Juni 2017 **Laure Pédèches** (Toulouse/Potsdam):

About some stochastic particle dynamics

In this talk, on the one hand, we will look into the asymptotic behaviour of stochastic version of the Cucker-Smale model, while also presenting a propagation of chaos result. On the other hand, we will give a result of existence and uniqueness for a particle system inspired by the Keller-Segel equation.

Di. 30. Juni 2017 **Hélène Guérin** (Rennes)

Ruin measures and occupation times for spectrally negative Lévy processes

I will present some risk measures for insurance models, the classical ruin and different parisian ruins, who are linked to occupation times of the surplus process. Some explicit results will be given in the specific case of spectrally negative Lévy processes in terms of scale functions.

Fr. 7. Juli 2017 **um 9:00 Vitalii Senin** (Berlin)

Pesin's formula for isotropic Brownian flows

Pesin's formula is a relation between the entropy of a dynamical system and its positive Lyapunov exponents. This formula was first established by Pesin in the late 1970s for some deterministic dynamical systems acting on a compact Riemannian manifold. Later were obtained plenty of generalizations of it. For example, different authors have proved the formula for some random dynamical systems, or have relaxed the condition of state space compactness. Nevertheless, it has never been obtained for dynamical systems with invariant measure, which is infinite. The problem is that if invariant measure is infinite, then the notion of entropy becomes senseless. Invariant measure of isotropic Brownian flows is the Lebesgue measure on R^d, which is, clearly, infinite. Nevertheless, we define entropy for such a kind of flows using their translation invariance. Then we study the analogue of Pesin's formula for these flows using the defined entropy.

Di. 18. Juli 2017 um 10:15 Tania Kosenkova (Potsdam)

Noise sensitivity of Lévy driven SDE's: *estimates and applications*

The topic of this talk is induced by the following question: whether the deviation between the solutions of two different Lévy driven SDE's can be controlled in terms of the characteristics of the underlying Lévy processes?
In the case of SDE's with additive noise we give the estimate for the deviation between the solutions in terms of the coupling distance for Lévy measures, which is based on the notion of the Wasserstein distance. In case of Lévy-type processes, whose characteristic triplets are state dependent, we exploit the fact that every Lévy kernel can be obtained by means of a certain infinite Lévy measure and the transform function. And under an appropriate set of conditions on the state dependent characteristic triplet the Lévy-type process can be described as a strong solution to a Lévy driven SDE with multiplicative noise. The estimate of the deviation between two Lévy-type processes is given in terms of transportation distance between the Lévy kernels,

which uses the transform functions of the kernels. Such estimates can be applied to the analysis of the low-dimensional conceptual climate models with paleoclimate data.

Ester Mariucci (HU Berlin)

Wasserstein distances between discretely observed Lévy processes

We present some upper bounds for the Wasserstein distance of order p between the product measures associated with the increments of two independent Lévy processes with possibly infinite Lévy measures. As an application, we derive an upper bound for the total variation distance between the marginals of two independent Lévy processes with possibly infinite Lévy measures and non-zero Gaussian components. Also, a lower bound for the Wasserstein distance of order p between the marginals of two independent Lévy processes is discussed. This is a joint work in progress with Markus Reiß.

Interessenten sind herzlich eingeladen !

Prof. Dr. Sylvie Roelly