Introduction to Stochastic Processes WS2018/19

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Exercises

1. Let $(Z_n)_{n\geq 0}$ describes a Bienaymé-Galton-Watson process with one ancestor and the reproduction rate of an individual given by probability distribution $\mathbf{r} = (r_j)_{j\geq 0}$. Find the probability generating function f and mean m of $\mathbf{r}$ and the probability generating function f_2 of Z_2, together with the probability to die out in second generation d_2, in the following cases:

   (a) $r_0 = 0,7,\ r_3 = 0,3$,

   (b) $r_j = \frac{1}{3}\left(\frac{2}{3}\right)^j, j \geq 0$.

2. Let us assume that we observe at time $t = 0$ a population of N cells. Each of them can either die at the end of the cycle or divide with probability $p \in (0, 1)$. For the process $(X_n)_{n\geq 0}$, that denotes the number of cells after n cycles find

   (a) the mean of $X_n, n \geq 0$,

   (b) conditions on p such that the population eventually survive and grow, and the survival probability,

   (c) threshold for p in which the survival probability is more than 95%.