Lecture course *Brownian Motion and the Feynman-Kac Formula on Riemannian manifolds*, WS 2021/2022, TU Chemnitz

The lecture course and the exercise sessions will be held via live zoom lectures, and can be held in ENGLISH AS WELL AS IN GERMAN, depending on what the participants prefer.

Lectures: Mondays 13:45 - 15:15, Zoom Meeting-ID: 853 9740 6906, Zoom Kenncode: 092301

Wednesdays 13:45 - 15:15, Zoom Meeting-ID: 863 6940 8939, Zoom Kenncode: 942687.

Exercises: Tuesdays 13:45 - 15:15, Zoom Meeting-ID: 891 7381 5733, Zoom Kenncode: 414618.

Prerequisites: A basic knowledge of functional analysis (spectral theorem) and manifolds (tangent spaces, vector fields, differential forms) will be helpful. If requested by the students, both topics can be briefly summarized in the beginning of the course.

Contents: In the first part of the course we are going to introduce the heat kernel on a Riemannian manifold M as a fundamental solution to the heat equation

$$\partial_t u(x,t) = \Delta u(x,t),$$

with Δ the Laplace-Beltrami operator on M, and to show the existence of Brownian motion on M as a diffusion process $X_t(x)$, $t \geqslant 0$, $x \in M$ on M having the heat kernel as its transition density with respect to the Riemannian volume measure. In the second part of the course, we are going to establish the famous *Feynman-Kac formula*, which states, given a potential $V : M \to \mathbb{R}$ and an initial value $\Psi_0 : M \to \mathbb{R}$, the solution $\Psi : M \times [0,\infty) \to \mathbb{R}$ of the initial value problem

$$\partial_t \Psi(x,t) = (\Delta - V(x))\Psi(x,t), \quad \Psi(\cdot,0) = \Psi_0$$

is given by the path integral formula

$$\Psi(x,t) = \int_{C([0,\infty),M)} e^{-\int_0^t V(\gamma(s))ds} \Psi_0(\gamma(t)) \mathrm{dP}^x(\gamma),$$

where P^x is the law of $X(x)$ (thus a probability measure on the space of continuous paths $C([0,\infty),M)$ on M which start in $x \in M$.
This formula is then used to explain some regularity results for molecular Schrödinger operators. If time admits, we will also establish the basics of stochastic (Ito-) integrals and explain the generalization of the Feynman-Kac formula to operators with magnetic fields, which is the celebrated *Feynman-Kac-Ito formula.*