Hidden symmetries and Maxwell fields on type-D vacuum spacetimes

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Introduction

- One of the major open issues in General Relativity is the **black hole stability problem**.
- Understanding the behavior of **linear fields** is the first most important step.
- The **Kerr solution** to Einstein equations describe the spacetime geometry of a rotating black hole in vacuum.
- To determine its physical relevance, we must study if it is stable against perturbations. Its stability under **Maxwell** and **linearized gravitational fields** remains an **open question**.

Introduction

- All stationary vacuum black holes (i.e the Kerr family) are **type D** in Petrov classification.
- These metrics admit an important object called a **Killing spinor**, which is responsible for the **separability properties** of several equations in Kerr spacetime.
- In **Minkowski**, Killing spinors serve to **generate solutions** of **higher spin field** equations from solutions of the **wave equation**.

Introduction

- In a recent work[1], the **complete (nonmodal) linear stability** of **Schwarzschild** under gravitational perturbations was proved by using a scalar variable which turns out to be $\Phi \sim \delta R_{\alpha\beta\gamma\delta} Y^{\alpha\beta *} Y^{\gamma\delta}$, with $Y_{\alpha\beta}$ a KY 2-form.
- The perturbed metric can be reconstructed **entirely** from the scalar variable Φ.
- The spinorial form of this variable is $\delta\psi_{ABCD} K^{AB} K^{CD}$, with K_{AB} a Killing spinor.
- In this work we will focus on the **Maxwell field**.

[1] **Dotti, Phys. Rev. Lett. 112 (2014) 191101 [arXiv:1307.3340 [gr-qc]]**

Introduction

We can classify black hole perturbations according to the spin of the fields:

- spin 0: scalar fields ϕ
- spin 1/2: Dirac fields χ_A
- **spin 1: Maxwell fields** ϕ_{AB}
- spin 2: gravitational fields ψ_{ABCD}

The (massless, free) field equations can be written in the form

$$\boxed{\nabla^{A_1' A_1} \phi_{A_1 \ldots A_n} = 0} \tag{1}$$

Example

The electromagnetic field $\mathbf{F}_{\mu\nu} = \phi_{AB}\bar{\epsilon}_{A'B'} + \bar{\phi}_{A'B'}\epsilon_{AB}$ satisfies the vacuum Maxwell equations $dF = 0 = d * F$, whose spinorial counterpart is

$$\nabla^{A'A} \phi_{AB} = 0. \tag{2}$$

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$$\nabla^{A'A}\phi_{AB} = 0. \qquad (2)$$

The geodesic problem

- Consider the geodesic problem $X^\mu \nabla_\mu X^\nu = 0$ in a curved space. We have

$$g_{\mu\nu} X^\mu X^\nu = \boldsymbol{\kappa} = \text{const.} \tag{3}$$

- Kerr spacetime is **stationary** and **axisymmetric**, so we have two isometries generated by the Killing fields

$$\boldsymbol{\xi}^\mu = (\partial_t)^\mu, \qquad \boldsymbol{\eta}^\mu = (\partial_\varphi)^\mu \tag{4}$$

and therefore we have two constants of motion

$$\mathbf{E} := -g_{\mu\nu}\xi^\mu X^\nu, \qquad \mathbf{L} := g_{\mu\nu}\eta^\mu X^\nu. \tag{5}$$

- As there are only two isometries, **it seems that we have not the required number of first integrals to solve the problem**.

The geodesic problem: Carter's constant

Remarkably, in Kerr spacetime there exists **another constant of motion**, originally discovered by Carter:

$$\mathbf{Q} \equiv H_{\mu\nu} X^{\mu} X^{\nu} \tag{6}$$

where $H_{\mu\nu} = H_{\nu\mu}$ is a **Killing tensor**: $\nabla_{(\sigma} H_{\mu\nu)} = 0$.

So, geodesic motion in Kerr is completely integrable

The Killing tensor is called a **hidden symmetry**, because it does not come from isometries of the manifold (it is irreducible in the sense that it cannot be expressed as a linear combination of products of Killing vectors).

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Klein-Gordon equation

- Consider now the **Klein-Gordon equation**, $\Box\psi = 0$. Replacing $X_\mu \to \nabla_\mu$, we have the operators

$$\mathbf{E} \to \hat{\xi} := \xi^\mu \nabla_\mu, \quad \mathbf{L} \to \hat{\eta} := \eta^\mu \nabla_\mu, \quad \boldsymbol{\kappa} \to \Box := \nabla_\mu g^{\mu\nu} \nabla_\nu, \tag{7}$$

acting on scalar fields, and these operators commutes between themselves:

$$[\hat{\xi}, \Box] = 0, \quad [\hat{\eta}, \Box] = 0, \quad [\hat{\xi}, \hat{\eta}] = 0. \tag{8}$$

- For the hidden symmetry: **Q** $\to \mathcal{Q} := \nabla_\mu H^{\mu\nu} \nabla_\nu$, and **an anomaly appears in the commutator**:

$$[\mathcal{Q}, \Box] = \tfrac{4}{3} \nabla_\nu ({R_\mu}^{[\sigma} H^{\nu]\mu}) \nabla_\sigma \tag{9}$$

- the anomaly disappears if the space is Einstein ($R_{\mu\nu} \sim g_{\mu\nu}$), or if $H_{\mu\nu}$ is the "square" of a **Killing-Yano tensor**: $H_{\mu\nu} = {Y_\mu}^\sigma Y_{\sigma\nu}$.

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Killing-Yano tensors

- A **Killing-Yano (KY) tensor** is a 2-form $Y_{\mu\nu} = -Y_{\nu\mu}$ that satisfies

$$\nabla_{(\sigma} Y_{\mu)\nu} = 0. \tag{10}$$

In Kerr spacetime we have indeed $H_{\mu\nu} = Y_{\mu}{}^{\sigma} Y_{\sigma\nu}$, and thus

$$[\mathcal{Q}, \Box] = 0. \tag{11}$$

Then, the **Klein-Gordon equation is completely separable in Kerr**.

- In spinor language, the KY tensor may be associated with a **Killing spinor** K_{AB} in the form

$$Y_{\mu\nu} = iK_{AB}\bar{\epsilon}_{A'B'} - i\bar{K}_{A'B'}\epsilon_{AB} \tag{12}$$

where $K_{AB} = K_{BA}$ and

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Killing spinors

The Killing spinor K_{AB} seems to be **the most primitive object** associated with the symmetries:

- $Y_{\mu\nu} = iK_{AB}\bar{\epsilon}_{A'B'} - i\bar{K}_{A'B'}\epsilon_{AB}$ is a **KY 2-form**

$$\nabla_{(\sigma}Y_{\mu)\nu} = 0. \tag{14}$$

- $H_{\mu\nu} = Y_{\mu}{}^{\sigma}Y_{\sigma\nu}$ is a **Killing tensor**

$$\nabla_{(\sigma}H_{\mu\nu)} = 0. \tag{15}$$

- $\xi^{A'A} := \nabla^{A'X}K_{X}{}^{A}$ is a **Killing vector**, $\xi \sim \partial_t$

$$\nabla_{(\mu}\xi_{\nu)} = 0. \tag{16}$$

- $\eta^{\mu} := H^{\mu}{}_{\nu}\xi^{\nu}$ is **another Killing vector**

$$\nabla_{(\mu}\eta_{\nu)} = 0. \tag{17}$$

Spin reduction in type D spacetimes

In a **type D spacetime** (e.g **Kerr** or **Schwarzschild**) there exists only one two-index Killing spinor, given by

$$\boxed{K_{AB} = \Psi_2^{-1/3} o_{(A}\iota_{B)}} \tag{18}$$

where o_A, ι_A is a principal dyad and Ψ_2 is the unique nontrivial Weyl scalar of the curvature.

- Let $\phi_{A_1 \ldots A_{2n}}$ be a spin s field (with integer spin), then it is not hard to prove that

$$(\Box + 2\Psi_2)(\phi_{A_1 \ldots A_{2n}} K^{A_1 A_2} \ldots K^{A_{2n-1} A_{2n}}) = 0 \tag{19}$$

- For instance, the Maxwell field ϕ_{AB} satisfies the **Fackerell-Ipser equation**

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Adjoint operators

Furthermore, **for any symmetric spinor field** ϕ_{AB} we have

$$\underbrace{\left[2K^{BC}\nabla_{A'C} + \tfrac{4}{3}(\nabla_{A'C}K^{BC})\right]}_{\mathcal{S}^{B}_{A'}} \underbrace{\nabla^{A'A}}_{\mathcal{E}^{A'A}} \phi_{AB} = \underbrace{(\Box + 2\Psi_2)}_{\mathcal{O}} \underbrace{K^{AB}}_{\mathcal{T}^{AB}} \phi_{AB} \tag{21}$$

namely:

$$\boxed{\mathcal{S}\mathcal{E}(\phi_{AB}) = \mathcal{O}\mathcal{T}(\phi_{AB})} \tag{22}$$

Now introduce an hermitian product and take the adjoint equation (**Wald**):

$$\mathcal{E}^{\dagger}\mathcal{S}^{\dagger}(f) = \mathcal{T}^{\dagger}\mathcal{O}^{\dagger}(f). \tag{23}$$

So, **a solution of the equation** $\mathcal{O}^{\dagger}(f) = 0$ **gives us a solution of** $\mathcal{E}^{\dagger}(\chi) = 0$.

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Symmetry operators

Theorem

Let $f : \mathcal{M} \to \mathbb{C}$ *be a solution of* $\mathcal{O}(f) := (\Box + 2\Psi_2) f = 0$*. Then,*

$$\phi_{AB} := \nabla_{A'(A}[\mathcal{S}^{\dagger}(f)]^{A'}_{B)} = -2\nabla_{B'(A}\left[\bar{K}^{A'B'}\nabla_{B)A'} f + \tfrac{1}{3}(\nabla_{B)A'}\bar{K}^{A'B'}) f\right] \tag{24}$$

is a solution of Maxwell equations, $\nabla^{A'A}\phi_{AB} = 0$*. Furthermore, the operator* $\mathcal{A}$ *defined by*

$$\mathcal{A}(f) = -2K^{AB}\nabla_{A'A}\left[\bar{K}^{A'B'}\nabla_{B'B} f + \tfrac{1}{3}(\nabla_{B'B}\bar{K}^{A'B'}) f\right] \tag{25}$$

maps solutions of $\mathcal{O}(f) = 0$ *in solutions:*

$$(\Box + 2\Psi_2)\mathcal{A}(f) = 0. \tag{26}$$

The Carter operator

- Remember the **Killing tensor** $H_{\mu\nu} = Y_{\mu}{}^{\sigma} Y_{\sigma\nu}$:

$$H_{\mu\nu} = -2K_{AB}\bar{K}_{A'B'} + \frac{1}{2}Re(\Psi_2^{-2/3})\epsilon_{AB}\bar{\epsilon}_{A'B'}, \tag{27}$$

and the **Carter operator**

$$\boxed{\mathcal{Q} = \nabla_{\mu} H^{\mu\nu} \nabla_{\nu}} \tag{28}$$

- Define now the complex tensor

$$P_{\mu\nu} := -2K_{AB}\bar{K}_{A'B'} + \frac{1}{2}\Psi_2^{-2/3}\epsilon_{AB}\bar{\epsilon}_{A'B'}, \tag{29}$$

and the complex operator

$$\boxed{\mathcal{P} := \nabla_{\mu} P^{\mu\nu} \nabla_{\nu}} \tag{30}$$

Note that **the real part of $P_{\mu\nu}$ is the Killing tensor**, $Re(P_{\mu\nu}) = H_{\mu\nu}$, and so **the Carter operator is $\mathcal{Q} = Re(\mathcal{P})$**.

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The Carter operator

Lemma

The operator $\mathcal{A}$ can be put in the form

$$\mathcal{A}(f) = \mathcal{P}(f) - \frac{1}{2}\Psi_2^{-2/3}(\Box + 2\Psi_2)f, \tag{31}$$

where $\mathcal{P}$ is defined as above. Thus, if f is a solution of $(\Box + 2\Psi_2)f = 0$, then $\mathcal{A} \equiv \mathcal{P}$.
In the case $\Psi_2 \in \mathbb{R}$, $\mathcal{P}$ is real and $\mathcal{A}$ equals the Carter operator.

Example

In **Schwarzschild**, $\Psi_2 \in \mathbb{R}$, so $\mathcal{A}$ is real. If f is a solution of $(\Box + 2\Psi_2)f = 0$, then $\mathcal{A}$ equals the Carter operator, which in turn agrees with the laplacian on the sphere, $\mathcal{A} = \mathcal{Q} \equiv \triangle_{S^2}$.

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Dirac fields

Consider now **Dirac fields**: $\gamma^\mu \nabla_\mu \psi := \nabla\!\!\!/ \psi = 0$. If we use $\mathcal{S}^B_{A'}$ (remember $\mathcal{SE}(\phi_{AB}) = \mathcal{OT}(\phi_{AB})$) to construct the operator

$$L_\alpha{}^\beta := \sqrt{2} \begin{pmatrix} 0 & \bar{\mathcal{S}}^{B'}_A - K_A{}^C \nabla^{B'}_C \\ \mathcal{S}^B_{A'} - \bar{K}_{A'}{}^{C'} \nabla^B_{C'} & 0 \end{pmatrix}, \tag{32}$$

then L is a **symmetry operator for the Dirac equation**:

$$\nabla\!\!\!/ \psi = 0 \quad \Rightarrow \quad \nabla\!\!\!/ L\psi = 0 \tag{33}$$

The operator L can be put in the form

$$L = -(\gamma_5 \gamma_\mu Y^{\mu\nu} \nabla_\nu + \tfrac{2}{3} \gamma_\mu \xi^\mu). \tag{34}$$

This last expression was already given by **Carter**, who found that L **commutes with the Dirac operator**.

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Conclusions

- The Killing spinor K_{AB} is the **most primitive object** associated with the symmetries.
- From solutions of the **scalar** equation $(\Box + 2\Psi_2)f = 0$, we can obtain solutions $\phi_{AB}(f)$ of **Maxwell** equations in a black hole spacetime.
- The operators we have found are intimately related with symmetries already known, such as the **Carter operator**.
- Can we construct all **physically interesting** Maxwell fields in black hole spacetimes from a scalar variable? If so, we can study the stability of the electromagnetic field through the analysis of a scalar equation.
- The problem of **linearized gravity** around a curved background **is not** the spin 2 system: $\nabla^{A'A}\delta\psi_{ABCD} \neq 0$. Can we extend our method to study this problem?

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