Supersymmetric black holes with single axial symmetry in five dimensions

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Introduction

Classification of black holes in $4D$

- 4D Einstein-Maxwell asymptotically flat solutions:
  - 3 parameter family of Kerr-Newman black holes (no-hair theorem [Israel 1967], [Hawking 1972], [Carter 1971], [Robinson 1975], [Bunting & Masood 1987], ...)
- 4D asymptotically flat supersymmetric (Einstein-Maxwell):
  - Majumdar-Papapetrou class [Chrusciel et al. 2006]
  - multiple extremal Reissner-Nordstrom black holes
- Possible generalisations: much less known
  - more general theories
  - multi-black holes
  - different asymptotics (Kaluza-Klein, AdS, dS)
  - higher dimensions

Higher dimensional black holes

Examples (vacuum):

- Schwarzschild-Tangherlini [Tangherlini 1963]: spherically symmetric

$$g = -(1 - \mu/r^{D-3})\mathrm{d}t^2 + (1 - \mu/r^{D-3})^{-1}\mathrm{d}r^2 + r^2\Omega^2_{D-2}$$

- Myers-Perry [Myers & Perry 1986]
  - vacuum black hole rotating in $N = \lfloor \frac{D-1}{2} \rfloor$ independent planes
  - $\mathbb{R} \times U(1)^N$ symmetry (generically), $N \geq 2$
- black rings ($D = 5$) [Emparan & Reall 2001]
  - horizon with $S^2 \times S^1$ topology
  - $\mathbb{R} \times U(1)^2$ symmetry
  - overlaps with asymptotic charges of MP $\implies$ no uniqueness

Landscape of higher dimensional black holes

Richer space of solutions

- horizon topology theorems [Hawking 1972], [Galloway & Schoen 2006]
  - less restrictive for $D > 4$
- smooth multi-black hole solutions
  e.g. black Saturn [Elvang & Figueras 2007]
- solitonic solutions (bubbling spacetimes) [Bena & Warner 2005]
- black holes in bubbling DOC [Kunduri & Lucietti 2014]
- **classification is an open problem**

Symmetries

- all solutions so far at least $\mathbb{R} \times U(1)^2$
- conjectured black holes with $\mathbb{R} \times U(1)$ symmetry [Reall 2003]
  - rigidity theorem requires $\mathbb{R} \times U(1)$ (for rotating black holes) [Hollands et al. 2007]
  - supported by approximate/numerical methods [Emparan et al. 2010], [Dias et al. 2010]

Supersymmetric solutions

Bosonic sector of $D = 5$ minimal supergravity: metric g, Maxwell field F

Definition

A solution $(\mathcal{M}, g, F)$ is supersymmetric if it admits a non-trivial solution of the Killing spinor equation $\tilde{\nabla}^{(g,F)}\epsilon = 0$.

- (g, F) invariant under supersymmetry transformation generated by ϵ
- saturate BPS bound $M \geq |Q|$ (for asymptotically flat) [Gibbons & Hull 1981]
- SUGRA as low-energy limit of string theory
- microscopic derivation of black hole entropy [Strominger & Vafa 1996]
- finding all black holes with given charges is crucial for entropy counting

Supersymmetric Black holes in $D = 5$

Near-horizon classification [Reall 2003]

- extremal horizon with local geometries: (squashed) S^3, $S^2 \times S^1$
- near-horizon geometry: $AdS_2 \times S^3$, $AdS_3 \times S^2$

BMPV solution: rotating, charged BH [Breckenridge et al 1997]

- horizon topology S^3 with (squashed) 3-sphere geometry

Black rings: $S^2 \times S^1$ horizon geometry [Elvang et al. 2004]
Black lens: $L(p, 1) = S^3/\mathbb{Z}_p$ [Kunduri & Lucietti 2014], [Tomizawa & Nozawa 2016], [Breunhölder & Lucietti 2017]
Black holes in 'bubbling spacetimes'

- $S > S_{BMPV}$ possible [Horowitz, Kunduri & Lucietti 2017]

Classification assuming $\mathbb{R} \times U(1)^2$ symmetry [Breunhölder & Lucietti 2017]
Construction and classification of black holes with $\mathbb{R} \times U(1)$ [DK & Lucietti 2022]

Local solution from Killing spinor

5D minimal SUGRA

Assume that $\widetilde{\nabla}^{(g,F)}\epsilon = 0$ has non-trivial solution.
Strategy: use Killing spinor bilinears [Gauntlett et al. 2003]

- scalar $f \sim \bar{\epsilon}\epsilon$
- causal vector field $V^a \sim \bar{\epsilon}\gamma^a\epsilon$
- three two-forms $X^{(i)}_{ab} \sim \bar{\epsilon}\gamma_{ab}\epsilon$, $i = 1, 2, 3$

Algebraic and differential identities:

- $g(V, V) = -f^2, \qquad \mathcal{L}_V g = \mathcal{L}_V F = 0$
- if $f \neq 0$, define base metric h on $\mathcal{M}/\mathbb{R}_V$:

$$g = -f^2(dt + \omega)^2 + f^{-1}h, \qquad V = \partial_t$$

- $(h, X^{(i)})$ is hyper-Kähler:
  $\nabla^{(h)}X^{(i)} = 0$, $X^{(i)} \cdot X^{(j)} = -\delta_{ij} + \epsilon_{ijk}X^k$

Assume existence of $U(1)$ Killing field W, commuting with supersymmetry ($\mathcal{L}_W\epsilon = 0$)

- orbit space metric h can be written in Gibbons-Hawking form ($W = \partial_\psi$):

$$h = H^{-1}(\mathrm{d}\psi + \chi)^2 + H\mathrm{d}x^i\mathrm{d}x^i,$$

$$\star_3\mathrm{d}\chi = \mathrm{d}H, \qquad \mathrm{d}\star_3\mathrm{d}H = 0$$

- The full solution $(\mathcal{M}, g, F)$ **locally** determined by four harmonic functions H, K, L, M on $\mathbb{R}^3$ [Gauntlett et al., 2003]

$$g = -f^2(\mathrm{d}t + \hat{\omega} + \omega_\psi(\mathrm{d}\psi + \chi))^2 + f^{-1}h$$

  f, ω_ψ, etc. can be expressed in terms of H, K, L, M
- We need a **global** analysis for asymptotically flat black holes

Classification Theorem

Theorem (K & Lucietti 2022)

A supersymmetric, asymptotically flat, axisymmetric black hole or soliton solution of $D = 5$ minimal supergravity must have a Gibbons-Hawking base and is globally determined by four associated harmonic functions that are of multi-centred type with simple poles at the centres, which correspond to horizon components or fixed points of the axial symmetry.

Furthermore, the solution is smooth in the DOC and at the horizon if and only if the parameters of harmonic functions satisfy a set of algebraic constraints.

$$g = -f^2(\mathrm{d}t + \hat{\omega} + \omega_\psi(\mathrm{d}\psi + \chi))^2 + \frac{1}{f}\left(\frac{1}{H}(\mathrm{d}\psi + \chi)^2 + H\mathrm{d}x^i\mathrm{d}x^i\right)$$

$$H = \sum_{i=1}^{N} \frac{h_i}{|\boldsymbol{x} - \boldsymbol{a}_i|} + h_0, \qquad \text{and similarly for } K, L, M.$$

Parameter constraints

$$\sum_{i=1}^{N} h_i = 1, \qquad m = -\frac{3}{2}\sum_{i=1}^{N} k_i$$

$$h_i m + \frac{3}{2}k_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{h_i m_j - m_i h_j + \frac{3}{2}(k_i l_j - k_j l_i)}{|\boldsymbol{a}_i - \boldsymbol{a}_j|} = 0 \qquad \forall i \in \{1, ..., N\}$$

For fixed points of W:

$$h_i = \pm 1, \quad l_i = -h_i k_i^2, \quad m_i = \frac{1}{2}k_i^3, \quad h_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{2k_i k_j - h_i(h_j k_i^2 - l_j)}{|\boldsymbol{a}_i - \boldsymbol{a}_j|} > 0,$$

For horizons $h_i \in \mathbb{Z}$ and

$$-h_i^2 m_i^2 - 3h_i k_i l_i m_i + h_i l_i^3 - 2k_i^2 m_i + \frac{3}{4}k_i^2 l_i^2 > 0.$$

Outline of proof

- Multi centred solutions with Gibbons-Hawking base
  - Relate H, K, L, M functions to global invariants $\implies$ globally defined except for horizon & fixed points
  - determine behaviour of H, K, L, M at the horizon (by matching to near-horizon analysis of [Reall 2003])
  - determine behaviour of H, K, L, M at fixed points (harmonic function theory on $\mathbb{R}^3$)
  - analyse orbit space $\mathcal{M}/(\mathbb{R} \times U(1)) \longleftrightarrow \mathbb{R}^3$

$$H = \sum_{i=1}^{N} \frac{h_i}{|\boldsymbol{x} - \boldsymbol{a}_i|} + h_0, \qquad \text{and similarly for } K, L, M.$$

- Smooth asymptotically flat black hole solution
  - need to check regularity at fixed points and horizons by locally expanding
  - need to check asymptotic conditions

Properties of solutions

- if centres $\boldsymbol{a}_i$ are collinear in $\mathbb{R}^3 \implies$ extra $U(1)$ symmetry
  - reproduces $\mathbb{R} \times U(1)^2$ classification by [Breunholder & Lucietti 2017]
- generically, no additional symmetry $\rightarrow$ first explicit construction single axially symmetric BHs for $D > 4$
- horizon topologies: S^3, $S^2 \times S^1$, $L(p, 1)$
- non-trivial DOC topology (non-contractible 2-cycles)

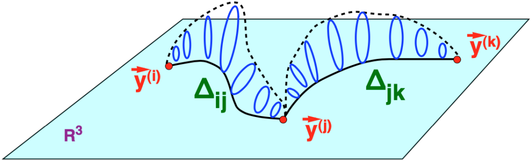

Figure: [Warner 2019]

- stationary Killing field null on time-like hypersurfaces in DOC ('evanescent ergosurfaces') [Niehoff & Reall 2016]

Summary and further directions

- We presented explicit method of constructing solutions with a single axial symmetry
- Provided a classification of such asymptotically flat black holes
- Further directions:
  - relaxing the condition $\mathcal{L}_W \epsilon = 0$
  - rigidity theorem does not apply (solutions with just stationary symmetry?)
  - different asymptotics: Kaluza-Klein [work in progress]
    - reduction to $4D$: generating and classifying $4D$ supersymmetric solutions
    - qualitative differences (supersymmetric Killing field might not agree with stationary KF)