Simplicial Complexes

Summer semester 2016 **Prof. Dr. M. Keller**

Sheet 9

(1) Let $k \geq 2$, and $G_0 = (V_0, E_0)$ be the graph induced by $\mathbb{Z}_k$, i.e., $V_1 = \mathbb{Z}_k$ and $E_0 = \{\{n, n+1\} \mid n \in \mathbb{Z}_k\}$. Let $G_1 = (V_1, E_1), \ldots, G_n = (V_n, E_n)$ be n copies of G_0 and for $m \in \mathbb{Z}_k$ let $G^{(m)} = (V^{(m)}, E^{(m)})$ be the graph such that we take union of $V_0, \ldots, V_n$ as vertex set while we identify the vertex 0 in every V_j and we identify the vertex m in every V_j, $j = 0, \ldots, n$. Let Δ be the associated simplicial complex. For which k and m is Δ a building?

(2) Let Σ be a Coxeter complex. Then the following statements are equivalent:

(i) Σ is finite.

(ii) $\mathcal{C}_\Sigma$ is finite.

(ii) $\mathrm{diam}(\Sigma)$ is finite.

(3) Let Δ be a building. Then the following statements are equivalent:

(i) Every apartment of Δ is finite.

(i) Every apartment of Δ has finite diameter.

(ii) $\mathrm{diam}(\Sigma)$ is finite.

(4) Let Δ be the simplicial complex associated to a tree. Show that there are infinitely many different systems of apartments such that Δ is a building.