Simplicial Complexes

Summer semester 2016 **Prof. Dr. M. Keller**

Sheet 6

(1) Let $(\Delta, \leq)$, $(\Delta', \leq)$ be chamber complexes, let $\varphi : \Delta \to \Delta'$ be a chamber morphism and let $\mathrm{typ}' : X_{\Delta'} \to I'$ be a coloring of Δ'.

(a) Show that $(\mathrm{typ}' \circ \varphi)|_{X_\Delta}$ is a coloring of Δ.

(b) Show that if $\mathrm{typ} : X_\Delta \to I$ is a coloring of Δ then there is a bijection $f : I \to I'$ such that

$$\mathrm{typ}' \circ \varphi = f \circ \mathrm{typ}.$$

(2) Draw A_2, A_3, C_2 and C_3, (for definition of A_n and C_n see example in lecture after Theorem 1.9).

(3) Show that the simplicial complex associated to the graph $G = (V, E)$ given by

$$V = \mathbb{Z} \times \{\pm 1\}, \quad E = \{\{(k, \sigma), (m, -\sigma)\} \mid k, m \in \mathbb{Z}, k \neq m, \sigma \in \{\pm 1\}\}$$

is isomorphic to the poset of cosets $\Delta(\mathbb{Z}^2, \{(1,0), (0,1)\})$. Make a drawing.

(4) Let G be a finitely generated group with minimal generating set S. Let $\delta(g, h) = \min\{i \mid g = s_1 \ldots s_i h, s_1, \ldots, s_n \in S \cup S^{-1}\}$ if $g \neq h$ and $\delta(g, g) = 0$.

(a) Show that δ is a metric.

(b) Assume such that $\Delta(G, S)$ is a simplicial complex. Show that

$$\delta(g, h) = d(gG_\emptyset, hG_\emptyset),$$

where d is the geodesic distance in $\Delta(G, S)$.