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## C\*-Algebras

Winter semester 2016/17

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### Sheet 9

- (1) Let  $b$  be a selfadjoint element of a  $C^*$ -algebra such that  $b \geq 0$ . Then

$$\sqrt{b_-} b_+ = 0,$$

where  $\sqrt{b_-}$  and  $b_+$  are defined via the continuous spectral calculus.

- (2) Show that for elements  $a, b$  in a  $C^*$ -algebra we have

$$\sigma(ab) \setminus \{0\} = \sigma(ba) \setminus \{0\}$$

- (3) Give an example of a maximal ideal in a  $C^*$ -algebra that is not closed.
- (4) Let  $(e_\iota)$  be a net of selfadjoint elements of a  $C^*$ -algebra. Then, the following statements are equivalent:

- (i)  $e_\iota a \rightarrow a$  for all  $a \in A$ .
- (ii)  $e_\iota a \rightarrow a$  for all selfadjoint  $a \in A$ .
- (iii)  $e_\iota a \rightarrow a$  for all  $a \in A$  with  $a \geq 0$ .
- (iv)  $a e_\iota \rightarrow a$  for all  $a \in A$ .
- (v)  $a e_\iota \rightarrow a$  for all selfadjoint  $a \in A$ .
- (vi)  $a e_\iota \rightarrow a$  for all  $a \in A$  with  $a \geq 0$ .

### Optional problem

- (O1) Show that the square function is not operator monotone, i.e., there is a  $C^*$ -algebra and elements  $0 \leq a \leq b$  but the inequality  $a^2 \leq b^2$  fails to hold.