C*-Algebras

Winter semester 2016/17 **Prof. Dr. M. Keller**

Sheet 7

(1) Let A be a commutative Banach algebra and I a maximal ideal. Show that the following statements are equivalent.

(a) I is regular.

(b) I is closed.

(Hint: You can of course use what was proven in the lecture.)

(2) Suppose that X is a compact Hausdorff space. If E is a closed subset of X, let ideal(E) be the ideal in $C(X)$ of functions which vanish on E. Furthermore, for a closed ideal J in $C(X)$ and let

$$E(J) = \bigcap_{f \in J} f^{-1}(\{0\}).$$

Show that

$$J = \mathrm{ideal}(E(J))$$

and that every ideal of $C(X)$ is of the form ideal(E) for some closed $E \subseteq X$. (Hint: Show that given an ideal J and an open neighborhood U of $E(J)$, then there is $f \in J$ such that $0 \leq f \leq 1$ and $f|_{X \setminus U} = 1$).

(3) Let X be a locally compact Hausdorff space and let $\overline{X}$ its one-point compactification, i.e. $\overline{X} = X \cup \{\infty\}$ with $U \subseteq \overline{X}$ open if and only if U is open in X or $\overline{X} \setminus U$ is compact in X. Show that $f \in C(X)$ belongs to $C_0(X)$ if and only if $\widetilde{f} \in C(\overline{X})$ with $\widetilde{f} = f$ on X and $\widetilde{f} = 0$ on ∞.

(4) Suppose that X is locally compact. Then every closed ideal in $C_0(X)$ is of the form

$$\{f \in C_0(X) \mid f|_E = 0\}$$

for some closed subset $E \subseteq X$.