C*-Algebras

Winter semester 2016/17 **Prof. Dr. M. Keller**

Sheet 5

(1) Let A be a Banach algebra without a unit. Let B be a unital Banach algebra and $i : A \to B$ a continuous injective map. Show that there is a unique continuous map $h : A_1 \to B$ such that

$$h \circ j = i$$

and

$$h(e) = e_B.$$

Is h also injective?

(2) Show that

$$f^*(n) = \overline{f(-n)}, \qquad n \in \mathbb{Z}$$

is an isometric involution the normed algebra $\ell^1(\mathbb{Z})$, but $\ell^1(\mathbb{Z})$ does not becomes an C^*-algebra.

(3) Give an example of an element a of a C^*-algebra such that

(a) $\sigma(a) \subseteq \mathbb{T}$ but $a^* \neq a^{-1}$,

(b) $\sigma(a) \subseteq \mathbb{R}$ but $a \neq a^*$.

(4) Let V be a normed vector space and $p : V \to \mathbb{C}$ be linear. Show that the following statements are equivalent:

(i) p is continuous

(ii) $\ker p := \{x \in V \mid px = 0\}$ is closed.