The lecture presents the interplay of analysis, dynamics, probability, spectral theory and mathematical physics in the realm of solid state physics. We seek to connect topological properties of dynamical systems such as the K-theory with spectral properties of the associated operators. Specifically, we aim to prove the Gap labeling theorem following the trace of Jean Bellissard. The first part of the lecture is devoted to measure-preserving dynamical systems, ergodicity and the pointwise ergodic theorem. This considerations become relevant in the study of ergodic random operators. We exhibit basic spectral properties, the (integrated) density of states for such random operators and the Pastur-Shubin trace formula. Then an introductory course in K-theory of C*-algebras follows focusing on C*-algebras defined through dynamical systems. With this at hand, we prove the Gap labeling theorem and discuss various explicit examples to compute the gap labels.

The second part of the lecture is devoted to more general structures than dynamical systems. So-called groupoids are introduced and it is shown how they can be used to describe models relevant in mathematical physics.

Required background

A solid background in the basic courses Analysis I-III, linear Algebra (in particular topology, measure theory, normed spaces (Banach spaces), Hilbert spaces (inner product)), functional analysis and spectral theory (spectral theorem for self-adjoint bounded operators) is required. Some background in C*-algebras will be helpful.

What can you learn?

- basic concepts in measure-preserving dynamical systems and ergodicity
- an introduction in ergodic theorems
- random operators and their spectral properties
- (integrated) density of states of random operators
- K-theory
- Gap labeling theorem
- groupoids and their role in solid state physics (in the discrete and continuous case)