Wolfgang Lück (Universität Münster):
The mystery of aspherical closed manifolds

A closed manifold is called aspherical if its universal covering is contractible. Such objects occur very often in interesting situations such as Riemannian manifolds with non-positive sectional curvature or irreducible 3-manifolds.

We want to discuss certain rigidity properties. A famous example is the Borel Conjecture which predicts that two closed aspherical manifolds are homeomorphic if and only if their fundamental groups are isomorphic. Another prominent conjecture is the generalization of the Hopf/Singer Conjecture that for aspherical closed manifolds the Euler characteristic satisfies a certain parity condition depending on the dimension and that its L^2-Betti numbers are concentrated in the middle dimension.

The question which groups do occur as fundamental groups of closed aspherical manifolds yields interesting connections to group homology.

Although some of these conjectures and questions have been solved in many cases, there is no satisfying explanation why the condition aspherical has all these consequences.