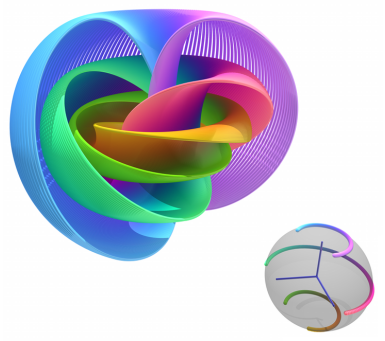
Blockseminar 2019
Harmonic Maps in Geometry

Christian Bär and Bernhard Hanke



Hopf fibration[1]

When:

Arrival: Sunday, June 23, 18:00 the latest
Departure: Friday, June 28, after lunch

Where:

Hotely Srní
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Czech Republic
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[1]Niles Johnson [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0)]

What:

Harmonic maps form a class of maps between Riemannian manifolds which contains various types of maps of great geometric interest: harmonic functions, geodesics, isometric minimal embeddings and many more. They are critical points of an energy functional and are characterized as solutions of a nonlinear elliptic partial differential equation. We will study existence, uniqueness, stability and regularity questions.

Once existence is established (which is not always the case, however), harmonic maps have important geometric applications. They provide information on the fundamental group, on the isometry group, on certain submanifolds and more.

How:

Presentations should not exceed 60 minutes (plus time for discussion). Please adhere to these times and take them into consideration when planning the lectures. For the case of time running short, be prepared to leave out passages in such a way that the rest of the lecture is not affected too much.

For lack of a blackboard, the lectures are held with two overhead projectors. It is important that the slides are not brought in prepared, but - as on a blackboard - are written live during the lecture.

If you are interested in attending please contact Hemanth Saratchandran (https://www.math.uni-augsburg.de/prof/diff/arbeitsgruppe/saratchandran). Provide a list of at least 3 talks, ordered by priority, of which you would like to present one. Participants are expected to stay for the whole event - no late arrivals and no early departures.

Program:

(0.) Introduction *(Christian Bär)*:

Presentation of the topic and overview

(1.) Energy, tension field, and first variation formula *(NN)*: [2, §3.1–§3.3]

(2.) Harmonic maps and the second variation formula *(NN)*: [2, §3.4–§3.5]

(3.) Examples of harmonic maps *(NN)*: [3, §4.3]

(4.) Instability *(NN)*: [3, §5.2]

(5.) Stability of holomorphic maps *(NN)*: [3, §5.3]

(6.) The heat flow method *(NN)*: [2, §4.1]

(7.) Existence of time-local solutions *(NN)*: [2, §4.2]

(8.) Existence of local time-dependent solutions *(NN)*: [2, §4.2]

(9.) Existence of global time-dependent solutions *(NN)*: [2, §4.3]

(10.) Applications: Theorems of Eells-Sampson and Hartman *(NN)*: [2, §4.4]

(11.) Applications: Theorem of Preissmann and complex submanifolds of Kähler manifolds *(NN)*: [2, §4.5 without Thm. 4.28], see also [1, §9.7]

(12.) Harmonic maps defined on surfaces *(NN)*: [1, §10.1]

(13.) Existence in 2 dimensions *(NN)*: [1, §10.2]

(14.) Regularity in 2 dimensions *(NN)*: [1, §10.3]

(15.) Harmonic maps to the unit sphere *(NN)*: Takahashi's theorem, doCarmo-Wallach theorem [3, pp. 189–191, 194–202]

(16.) Calabi's theorem *(NN)*: [3, pp. 203–207]

Literature:

[1] Jürgen Jost, *Riemannian geometry and geometric analysis*, 7. edition, Universitext, Springer, Cham, 2017.

[2] Seiki Nishikawa, *Variational problems in geometry*, Translations of Mathematical Monographs, vol. 205, American Mathematical Society, Providence, RI, 2002.

[3] Hajime Urakawa, *Calculus of variations and harmonic maps*, Translations of Mathematical Monographs, vol. 132, American Mathematical Society, Providence, RI, 1993.