The Arnoldi process

input: A, start vector q
normalise $q_1 = \frac{q}{\|q\|_2}$
for $k = 1$ to $m - 1$ **do**
  $q_{k+1} \leftarrow Aq_k$
  for $j = 1$ to k **do**
    $h_{jk} \leftarrow q_j^* q_{k+1}$
    $q_{k+1} \leftarrow q_{k+1} - q_j h_{jk}$
  end for
  reorthogonalise
  $h_{k+1,k} \leftarrow \|q_{k+1}\|_2$
  if $h_{k+1,k} = 0$ **then**
    span$\{q_1, \ldots, q_k\}$ is invariant under A
  end if
  $q_{k+1} = \frac{q_{k+1}}{h_{k+1,k}}$
end for
output: Hessenberg matrix H_m and orthonormal matrix Q_m

The Lanczos process

input: A, start vector q
normalise $q_1 = \frac{q}{\|q\|_2}$
for $k = 1$ to $m - 1$ **do**
  $q_{k+1} \leftarrow Aq_k$
  $\alpha_k = q_k^* q_{k+1}$
  $q_{k+1} \leftarrow q_{k+1} - \alpha_k q_k$
  if $k > 1$ **then**
    $q_{k+1} \leftarrow q_{k+1} - q_{k-1}\beta_{k-1}$
  end if
  $\beta_k \leftarrow \|q_{k+1}\|_2$
  if $\beta_k = 0$ **then**
    span$\{q_1, \ldots, q_k\}$ is invariant under A
  end if
  $q_{k+1} = \frac{q_{k+1}}{\beta_k}$
end for
output: tridiagonal matrix T_m and orthonormal matrix Q_m