

Challenges in Data Assimilation for Numerical Weather Prediction

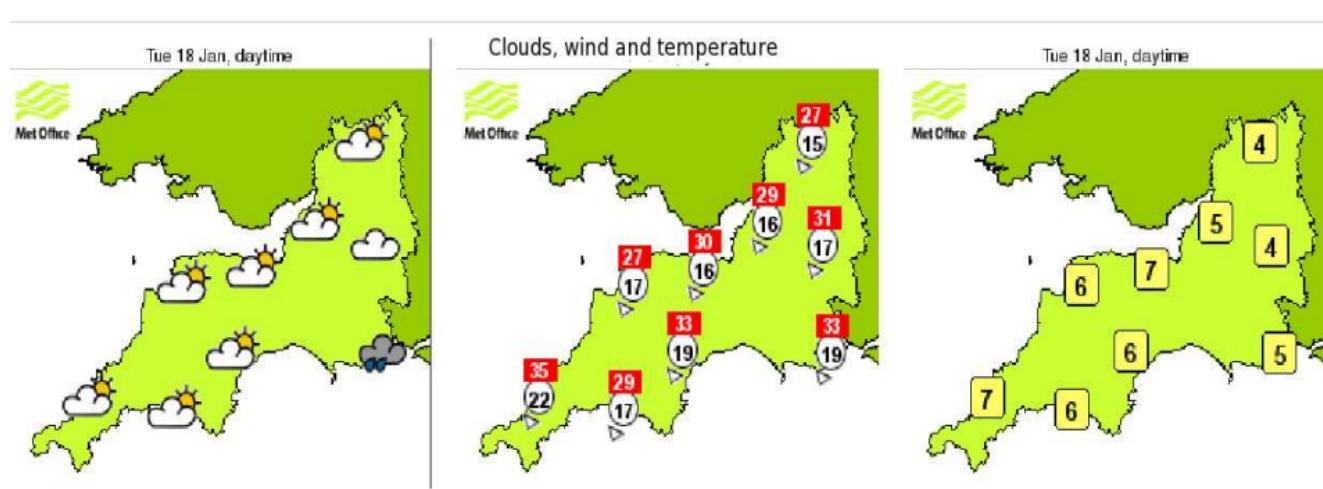
Melina Freitag

Department of Mathematical Sciences
University of Bath

2011 Vice-Chancellor's Research Day, 18th January 2011



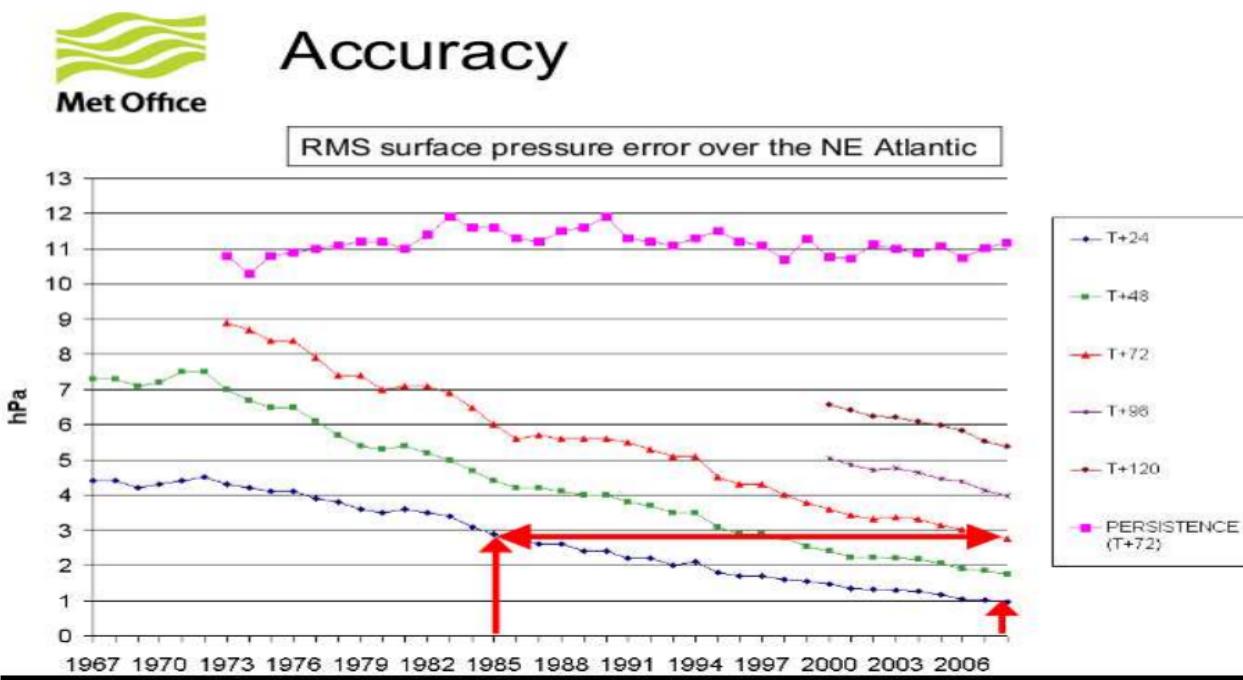
The MetOffice weather forecast for today (18/01/2011)



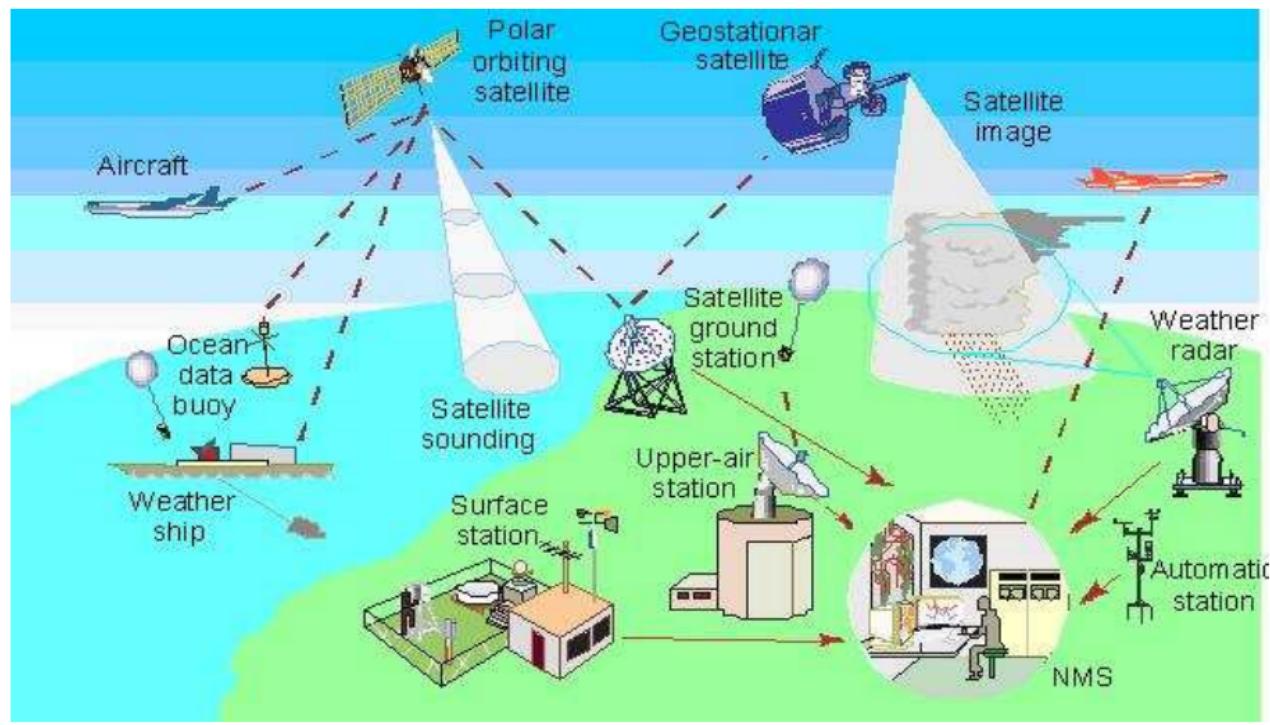
Forecast Bath: White Cloud, Temperature $0^{\circ} - 4^{\circ}$ Celsius, Wind 17 mph.



The UK MetOffice forecast over the last 40 years



Observation network



Global model

Introduce a 3D grid covering the atmosphere:



Global model

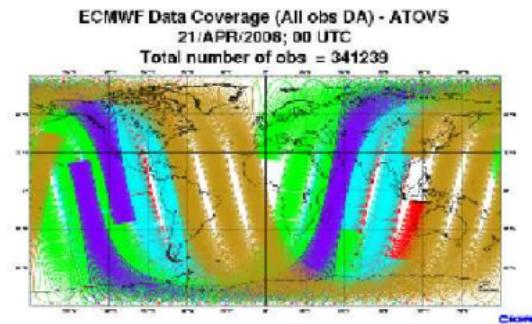
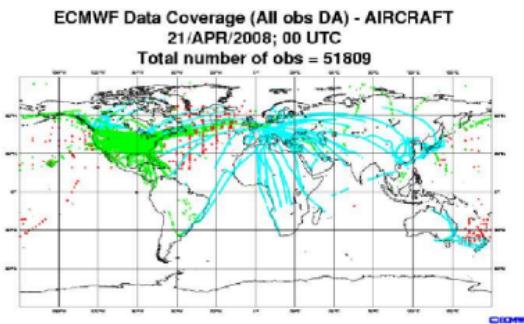
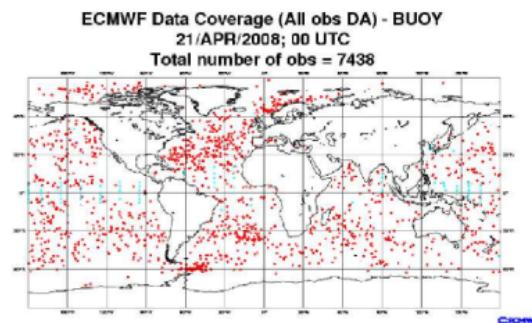
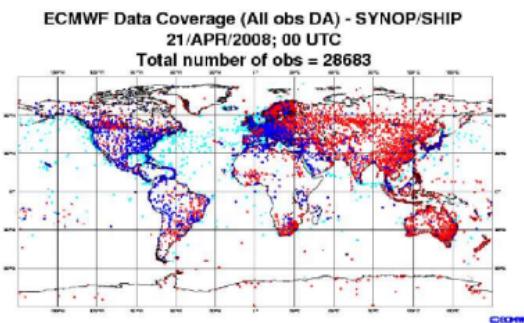
Introduce a 3D grid covering the atmosphere:



- In each of the $432 \times 320 \times 50$ grid points we have 7 variables (pressure, humidity, temperature, wind speed).
- Size of the state vector \mathbf{x} : $432 \times 320 \times 50 \times 7 \approx 10^7$.



Observations



Observations and state vector

- We put all the observations into a vector \mathbf{y} (size $\approx 10^5 - 10^6$).
- Size of the state vector \mathbf{x} : $432 \times 320 \times 50 \times 7 \approx 10^7$.
- $\mathbf{y} = H(\mathbf{x})$ maps from state space into observation space.
- Problem is **under-determined** and the observations are very **irregular**.

Wanted: estimate \mathbf{x}_i (time i) for the true atmospheric state

Observations \mathbf{y}_i

Satellites, ships and buoys,
surface stations, aeroplanes



Wanted: estimate \mathbf{x}_i (time i) for the true atmospheric state

Model

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

Observations \mathbf{y}_i

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model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

link between model and observation
space $\mathbf{y}_i = H(\mathbf{x}_i)$

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A priori information \mathbf{x}_i^B

Background state (previous
forecast)



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Assimilation algorithms

- find an (approximate) state of the atmosphere \mathbf{x}_i at time i
- forecast future states of the atmosphere

Wanted: estimate \mathbf{x}_i (time i) for the true atmospheric state

Model has errors!

model for the atmosphere

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Schematics of Data Assimilation (in 1D)

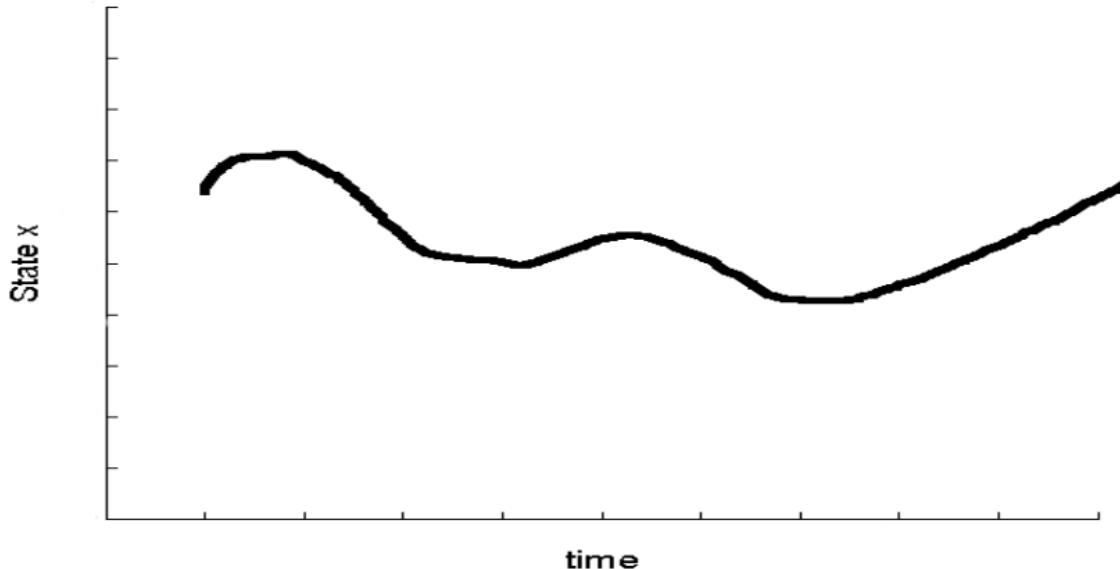


Figure: Previous forecast \mathbf{x}^B

Schematics of Data Assimilation (in 1D)

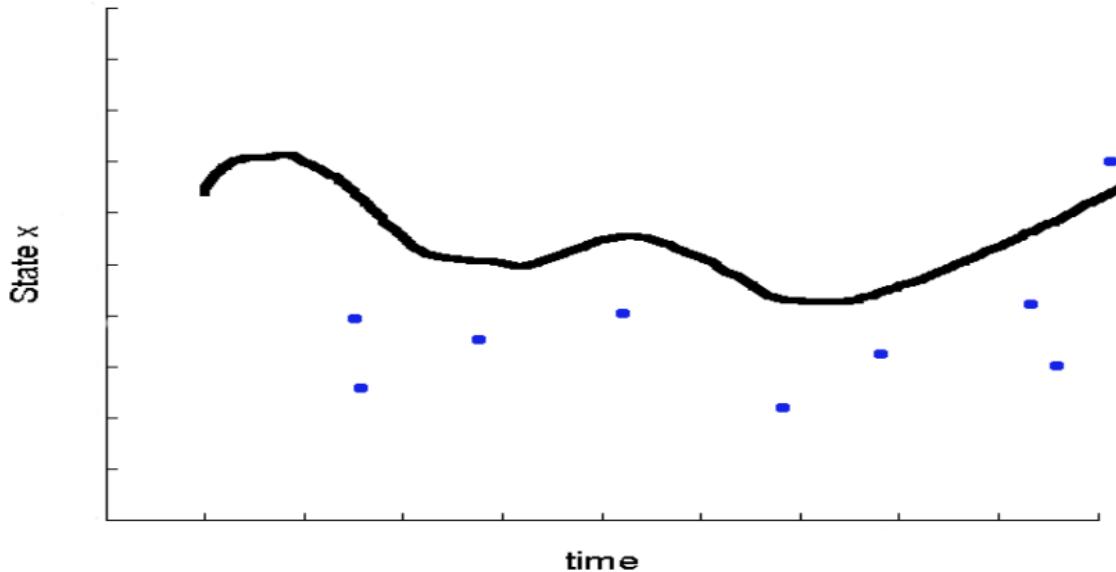


Figure: Observations y



Schematics of Data Assimilation (in 1D)

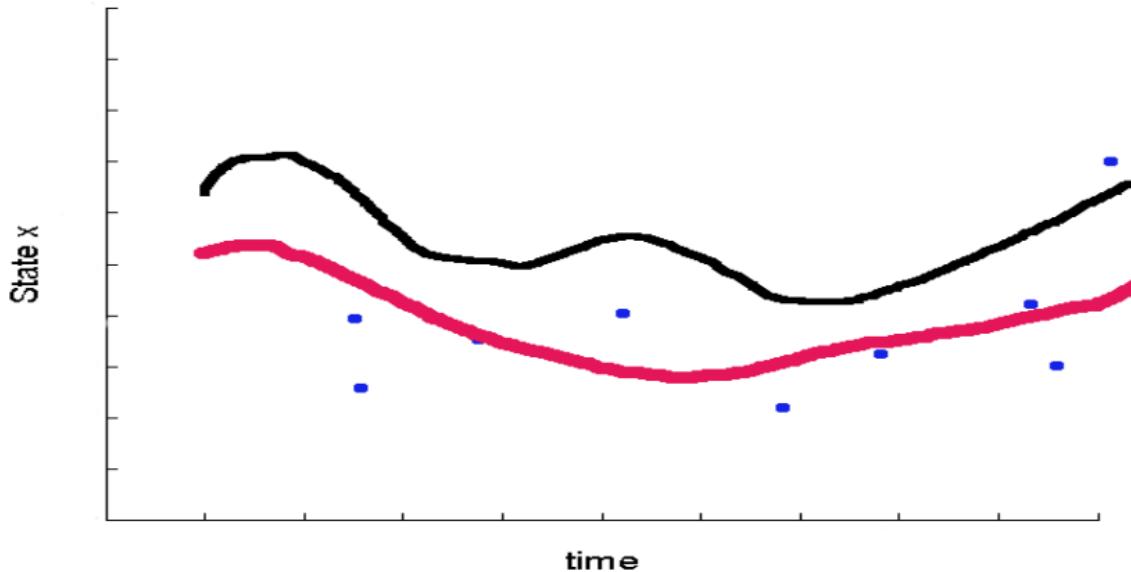


Figure: Analysis \mathbf{x}^A (consistent with observations and model dynamics)

Four-dimensional variational assimilation (4DVar)

Minimise the cost function

$$J(\mathbf{x}_0) = (\mathbf{x}_0 - \mathbf{x}_0^B)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^B) + \sum_{i=0}^n (\mathbf{y}_i - H(\mathbf{x}_i))^T \mathbf{R}_i^{-1} (\mathbf{y}_i - H(\mathbf{x}_i))$$

subject to model dynamics $\mathbf{x}_i = M_{0 \rightarrow i} \mathbf{x}_0$.

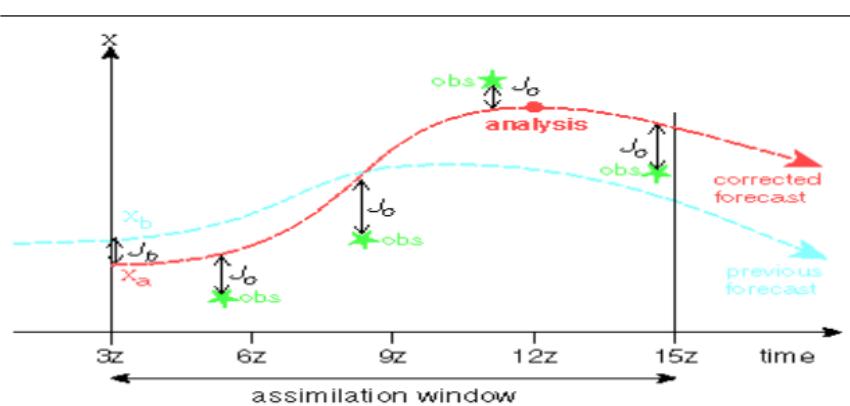


Figure: Copyright:ECMWF

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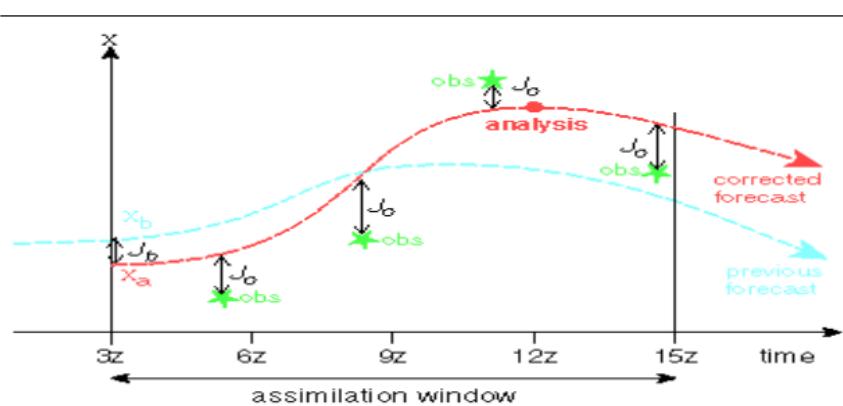


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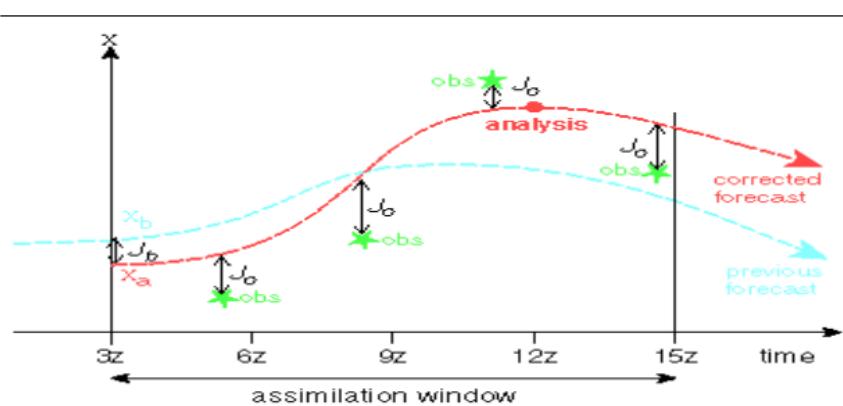


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Inverse Problems

Data Assimilation belongs to the class of **Inverse Problems**

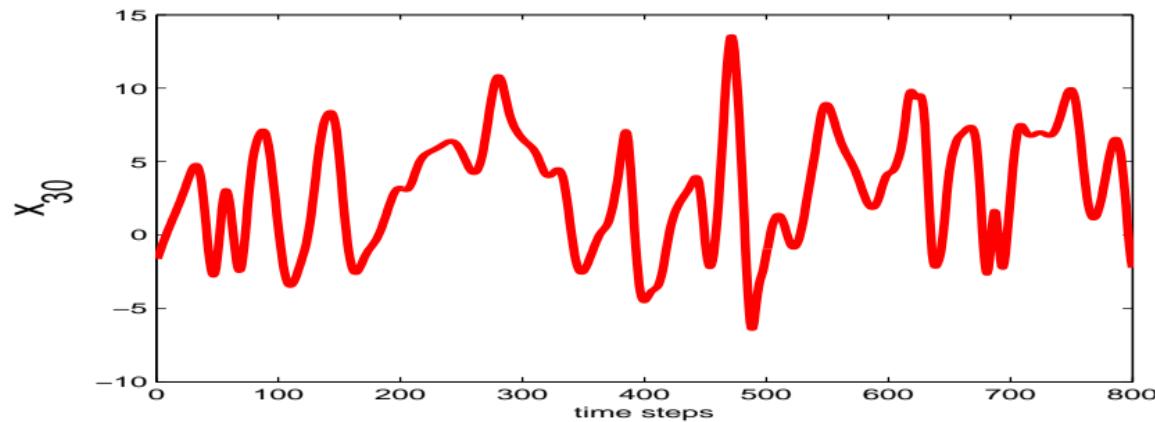


Figure: Solution to a "chaotic" problem

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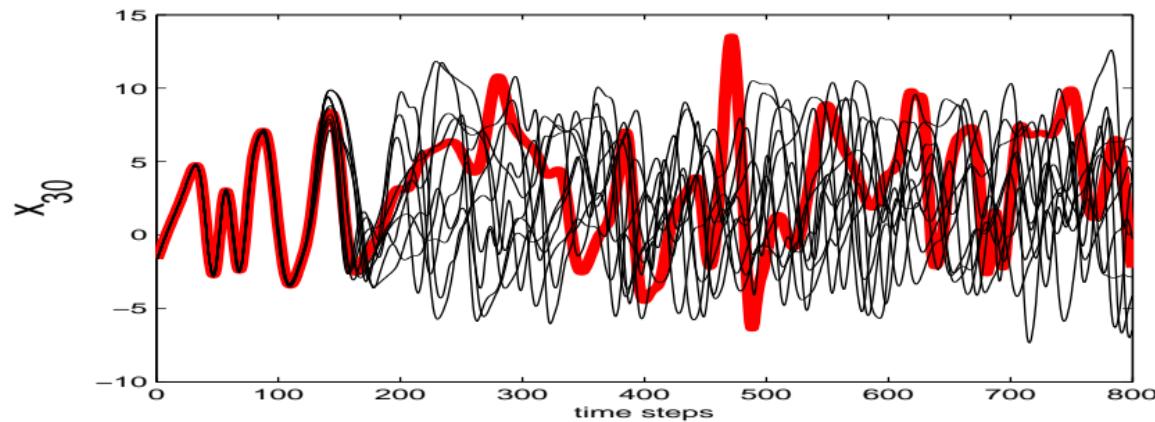
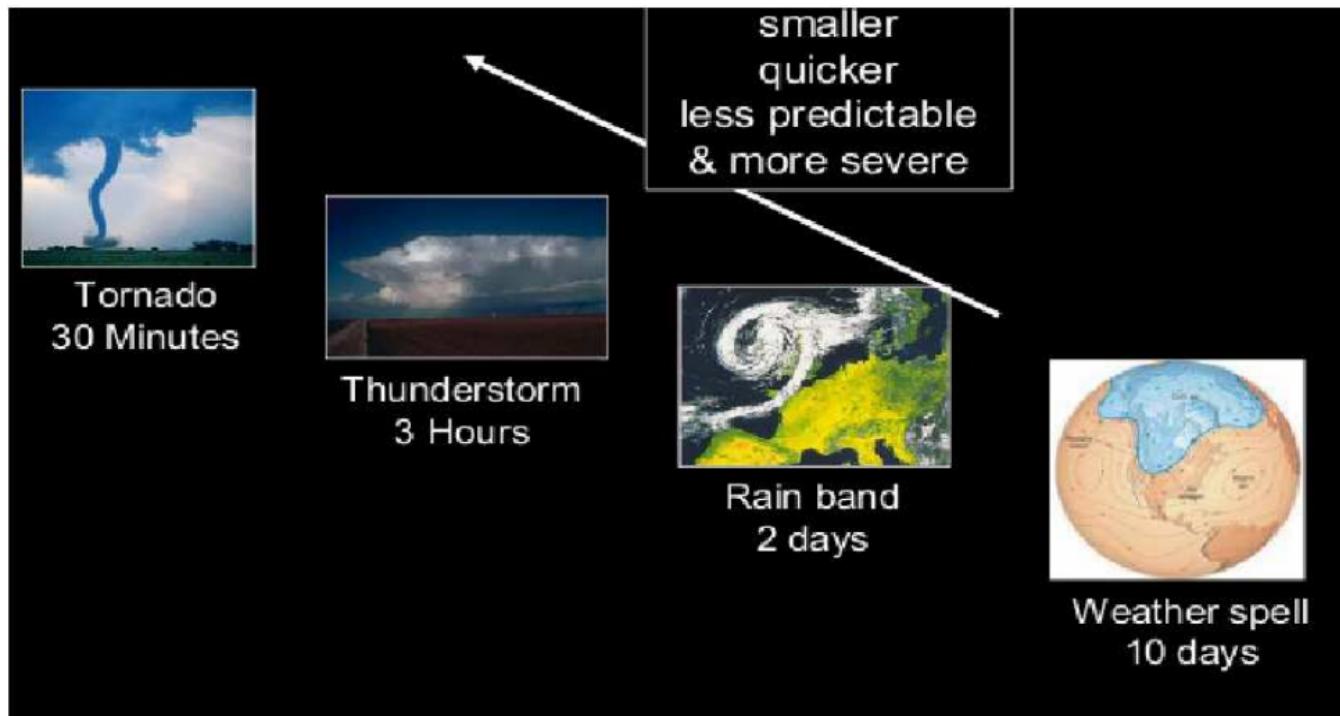


Figure: Solution to a "chaotic" problem

Resolution



Challenges in Data Assimilation

- forecast needs to be improved at smaller "spacial scales" and "time scales" in order to **forecast severe weather events**
- increasing the **model resolution** leads to computational challenges - **efficient implementations** of Data Assimilation algorithms
- several components of the Data Assimilation process need to be improved
 - model error
 - estimating the correct covariance matrices
 - **using better algorithms** in order to forecast sharp fronts