

# Challenges in Data Assimilation for Numerical Weather Prediction

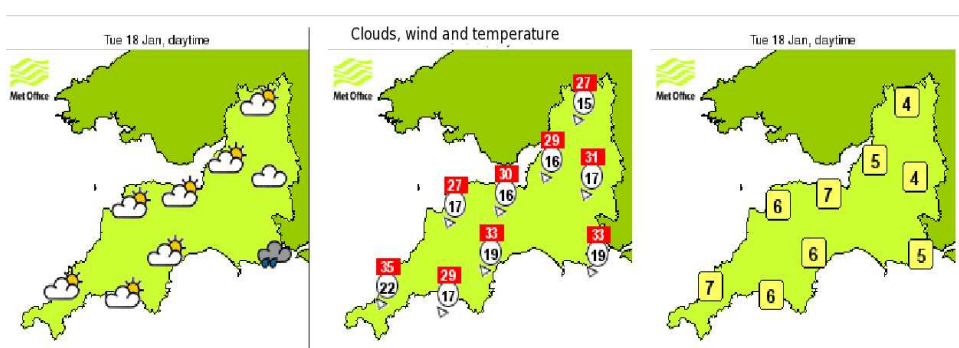
Melina Freitag

Department of Mathematical Sciences  
University of Bath

2011 Vice-Chancellor's Research Day, 18th January 2011

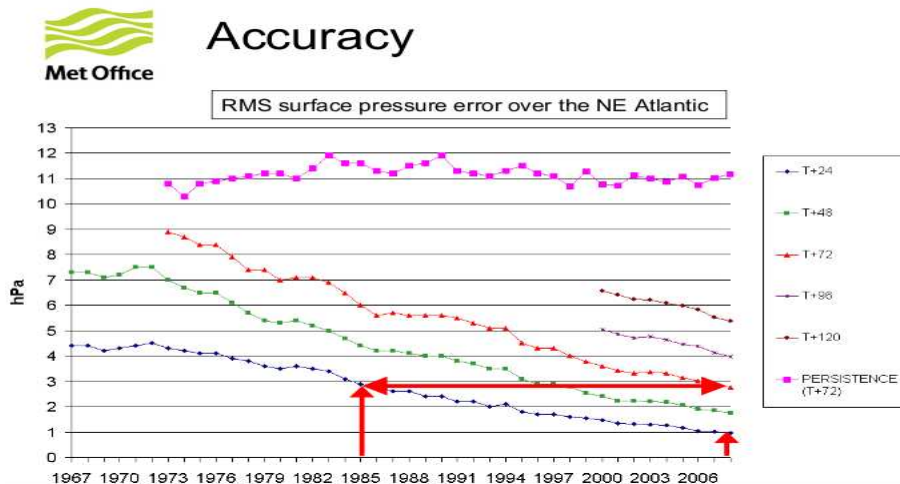


# The MetOffice weather forecast for today (18/01/2011)

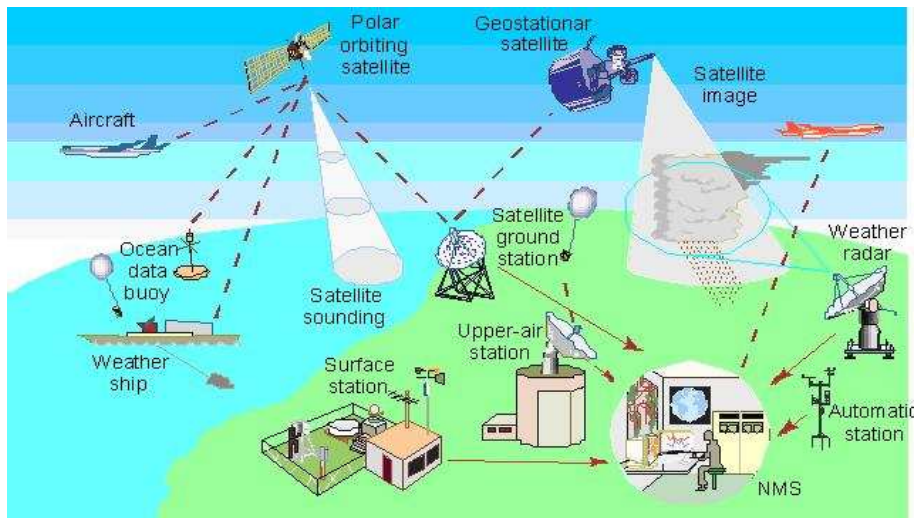


Forecast Bath: White Cloud, Temperature 0° - 4° Celsius, Wind 17 mph.

# The UK MetOffice forecast over the last 40 years

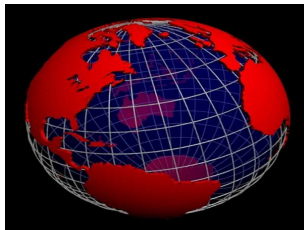


# Observation network



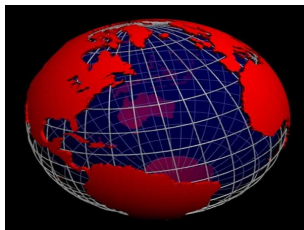
# Global model

Introduce a 3D grid covering the atmosphere:



# Global model

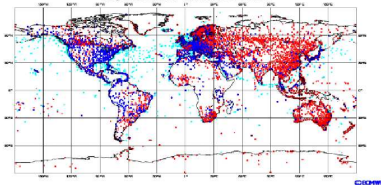
Introduce a 3D grid covering the atmosphere:



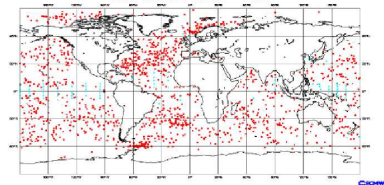
- In each of the  $432 \times 320 \times 50$  grid points we have 7 variables (pressure, humidity, temperature, wind speed).
- Size of the state vector  $\mathbf{x}$ :  $432 \times 320 \times 50 \times 7 \approx 10^7$ .

# Observations

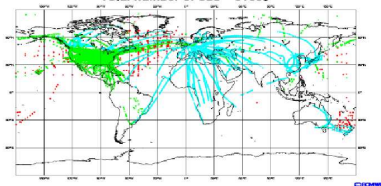
ECMWF Data Coverage (All obs DA) - SYNOP/SHIP  
21/APR/2008; 00 UTC  
Total number of obs = 28683



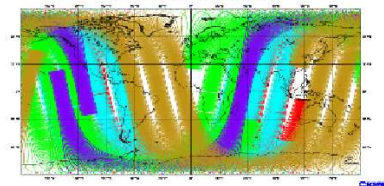
ECMWF Data Coverage (All obs DA) - BUOY  
21/APR/2008; 00 UTC  
Total number of obs = 7438



ECMWF Data Coverage (All obs DA) - AIRCRAFT  
21/APR/2008; 00 UTC  
Total number of obs = 51809



ECMWF Data Coverage (All obs DA) - ATOVS  
21/APR/2008; 00 UTC  
Total number of obs = 341239



# Observations and state vector

- We put all the observations into a vector  $\mathbf{y}$  (size  $\approx 10^5 - 10^6$ ).
- Size of the state vector  $\mathbf{x}$ :  $432 \times 320 \times 50 \times 7 \approx 10^7$ .
- $\mathbf{y} = H(\mathbf{x})$  maps from state space into observation space.
- Problem is **under-determined** and the observations are very **irregular**.



Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

Observations  $\mathbf{y}_i$

Satellites, ships and buoys,  
surface stations, aeroplanes

Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

### Model

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

### Observations $\mathbf{y}_i$

Satellites, ships and buoys,  
surface stations, aeroplanes

Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

### Model

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

link between model and observation  
space  $\mathbf{y}_i = H(\mathbf{x}_i)$

### Observations $\mathbf{y}_i$

Satellites, ships and buoys,  
surface stations, aeroplanes

Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

### Model

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

link between model and observation space  $\mathbf{y}_i = H(\mathbf{x}_i)$

### Observations $\mathbf{y}_i$

Satellites, ships and buoys,  
surface stations, aeroplanes

### A priori information $\mathbf{x}_i^B$

Background state (previous  
forecast)

Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

### Model

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

link between model and observation space  $\mathbf{y}_i = H(\mathbf{x}_i)$

### Observations $\mathbf{y}_i$

Satellites, ships and buoys, surface stations, aeroplanes

### A priori information $\mathbf{x}_i^B$

Background state (previous forecast)

### Assimilation algorithms

- find an (approximate) state of the atmosphere  $\mathbf{x}_i$  at time  $i$
- forecast future states of the atmosphere

Wanted: estimate  $\mathbf{x}_i$  (time  $i$ ) for the true atmospheric state

### Model has errors!

model for the atmosphere

$$\mathbf{x}_{i+1} = M(\mathbf{x}_i)$$

link between model and observation space  $\mathbf{y}_i = H(\mathbf{x}_i)$

### Observations $\mathbf{y}_i$ have errors!

Satellites, ships and buoys,  
surface stations, aeroplanes

### A priori information $\mathbf{x}_i^B$

Background state (previous  
forecast) has errors!

## Assimilation algorithms

- find an (approximate) state of the atmosphere  $\mathbf{x}_i$  at time  $i$
- forecast future states of the atmosphere

# Schematics of Data Assimilation (in 1D)

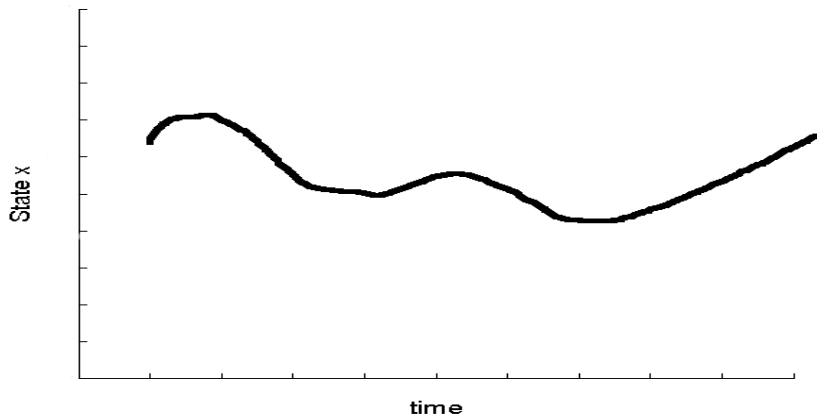


Figure: Previous forecast  $\mathbf{x}^B$

# Schematics of Data Assimilation (in 1D)

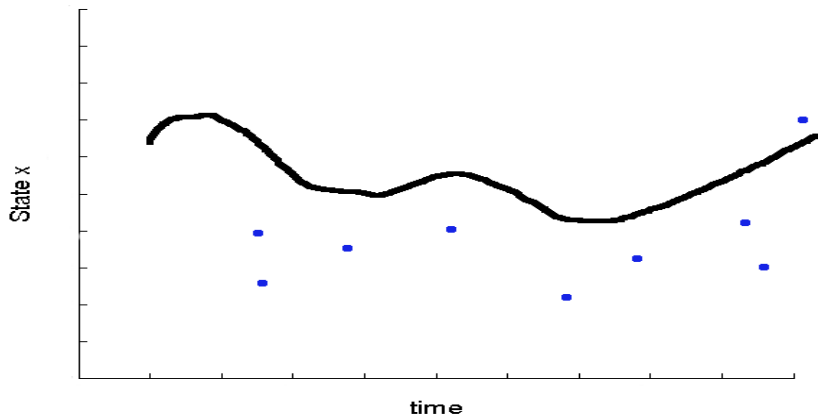


Figure: Observations  $y$



# Schematics of Data Assimilation (in 1D)

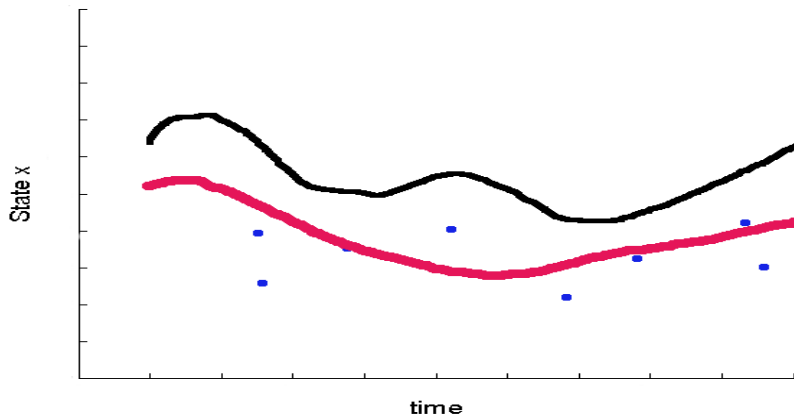


Figure: Analysis  $x^A$  (consistent with observations and model dynamics)

# Four-dimensional variational assimilation (4DVar)

Minimise the cost function

$$J(\mathbf{x}_0) = (\mathbf{x}_0 - \mathbf{x}_0^B)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^B) + \sum_{i=0}^n (\mathbf{y}_i - H(\mathbf{x}_i))^T \mathbf{R}_i^{-1} (\mathbf{y}_i - H(\mathbf{x}_i))$$

subject to model dynamics  $\mathbf{x}_i = M_{0 \rightarrow i} \mathbf{x}_0$ .

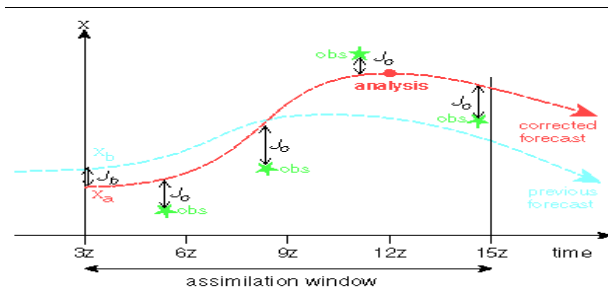


Figure: Copyright:ECMWF

# Four-dimensional variational assimilation (4DVar)

Minimise the cost function

$$J(\mathbf{x}_0) = (\mathbf{x}_0 - \mathbf{x}_0^B)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^B) + \sum_{i=0}^n (\mathbf{y}_i - H(\mathbf{x}_i))^T \mathbf{R}_i^{-1} (\mathbf{y}_i - H(\mathbf{x}_i))$$

subject to model dynamics  $\mathbf{x}_i = M_{0 \rightarrow i} \mathbf{x}_0$ .

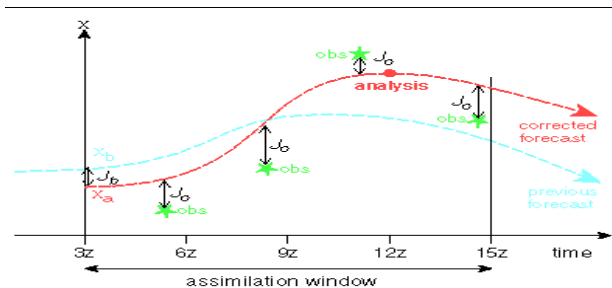


Figure: Copyright:ECMWF

# Four-dimensional variational assimilation (4DVar)

Minimise the cost function

$$J(\mathbf{x}_0) = (\mathbf{x}_0 - \mathbf{x}_0^B)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^B) + \sum_{i=0}^n (\mathbf{y}_i - H(\mathbf{x}_i))^T \mathbf{R}_i^{-1} (\mathbf{y}_i - H(\mathbf{x}_i))$$

subject to model dynamics  $\mathbf{x}_i = M_{0 \rightarrow i} \mathbf{x}_0$ .

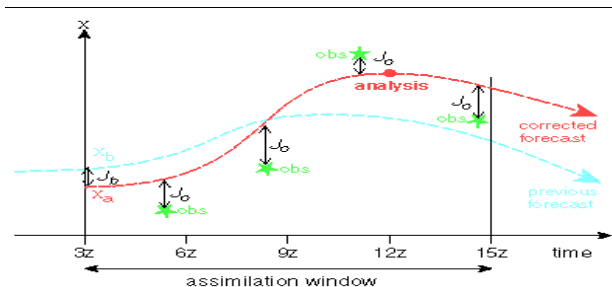


Figure: Copyright:ECMWF

# Inverse Problems

Data Assimilation belongs to the class of **Inverse Problems**

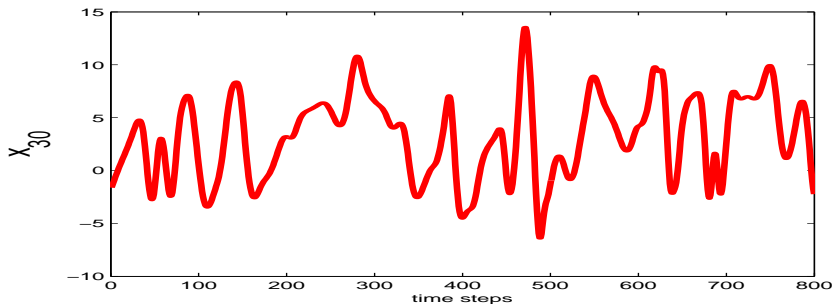


Figure: Solution to a "chaotic" problem

# Inverse Problems

Data Assimilation belongs to the class of **Inverse Problems**

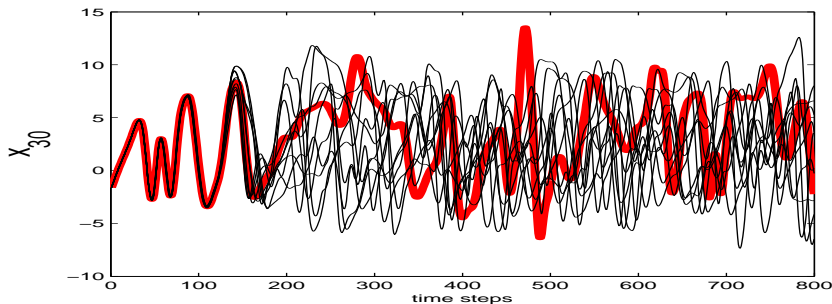
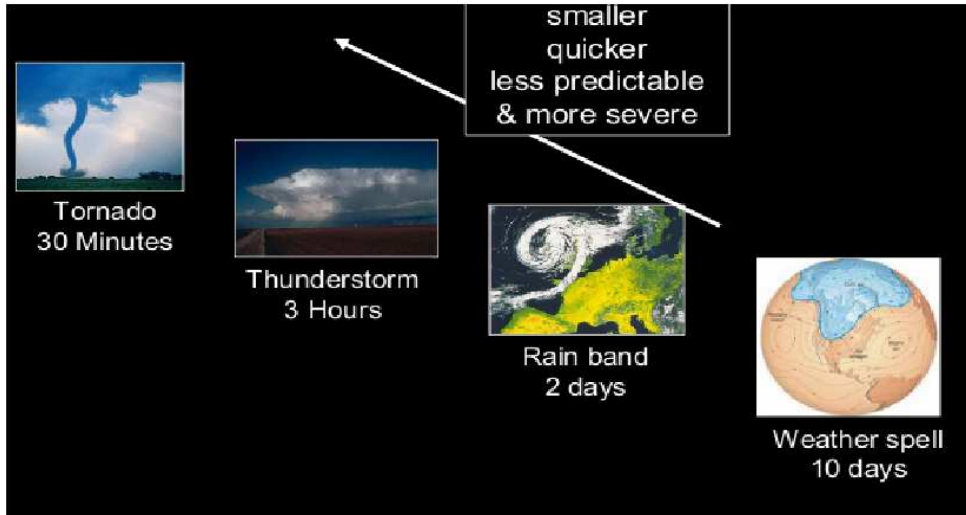


Figure: Solution to a "chaotic" problem

# Resolution



# Challenges in Data Assimilation

- forecast needs to be improved at smaller "spacial scales" and "time scales" in order to **forecast severe weather events**
- increasing the **model resolution** leads to computational challenges - **efficient implementations** of Data Assimilation algorithms
- several components of the Data Assimilation process need to be improved
  - model error
  - estimating the correct covariance matrices
  - using better algorithms in order to forecast sharp fronts