Perales - Convergence of Alexa spaces. 30/01/2016.

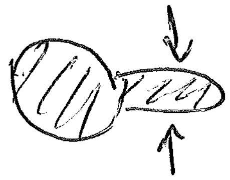
- GH convergence.
- IF convergence.

Def^n. (X, d_X), (Y, d_Y) compact.

$$d_{GH}((X,d_X),(Y,d_Y)) = \inf \{ d_H^Z(\varphi(X), \Phi(Y)) : \varphi : X \to Z,\ \Phi : Y \to Z \text{ isometrically} \}.$$

Def^n. (X, d_X, T_X), (Y, d_Y, T_Y). n-dim int current spaces.

$$d_F((X,d_X,T_X),(Y,d_Y,T_Y)) = \inf \{ d_F^Z(\varphi_\# T_X, \Phi_\# T_Y) : \varphi : X \to Z,\ \Phi : Y \to Z \text{ isometric embeddings} \}$$

Example: $X_j :=$  in $\mathbb{R}^2$. length distance.

$X_j \xrightarrow{GH}$  $X_j \xrightarrow{IF}$ 

See thus $GH \neq IF$.

Sormani-Wenger, Munn, Matveev-Portegies: $GH = IF$.
when having Ricci lower bound & non-collapsing.

Remarks: • GH distance doesn't preserve ~~distance~~ dimension (Haus).

- It is not stable, $M_j \xrightarrow{GH} X$ but X is not n-mfld.
- IF converges "preserves dimension"

$(X_j, d_j, T_j) \to (X, d, T)$ either 0 current
or n-int current sp.

Rmk (M,g), $T(\omega)=\int_M \omega$ integration of top forms.

Def^{n}. Currents (Ambrosio–Kirchheim). Z met space.

$T: \mathcal{D}^n(Z) := \mathrm{Lip}_b(Z) \times \mathrm{Lip}(Z)^n \to \mathbb{R}$, multilinear.

Lip_b bounded

s.t. T satisfies the following:

I) $T(f,\pi)=0$ if $\exists\, U$ open for which $\pi_i(x)=0$, $\forall x \in U$ and $f \neq 0$ in U.

II) T is cts w.r.t. π_i^j.

III) $\exists\, \mu$ finite Borel fcn s.t. $\forall (f,\pi)$

$$|T(f,\pi)| \leq \prod_{i=1}^{n} \mathrm{Lip}(\pi_i) \int |f| \, d\mu .$$

The smallest μ is denoted by $\|T\|$ (measure of T)

Ex. $A \subset \mathbb{R}^n$ Borel, $\theta \in L^1(A,\mathbb{R})$.

$[[\theta]] : \mathcal{D}^n(\mathbb{R}^n) \to$.

$$[[\theta]](f,\pi) = \int_A \theta \cdot f \cdot |\nabla\pi_1, \dots, \nabla\pi_n| \, d\mathcal{L}^n .$$

Def^{n}. Pushforward $\varphi: Z \to W$ Lipschitz, $\varphi_\# T(f,\pi) = (f\circ\varphi, \pi_1\circ\varphi, \dots, \pi_n\circ\varphi)$.

Boundary operator: $T \in \mathcal{D}^n(Z) \to \mathbb{R}$, $\partial T : \mathcal{D}^{n-1}(Z) \to \mathbb{R}$.

$$\partial T(f,\pi) := (1, f, \pi_1, \dots, \pi_{n-1}).$$

$(\partial(\partial T) = 0$ since condition (I) in def^{n}).

Def^{n} (Integral current)

Let T be an n-dim. current on Z, then T is an integral current if $\exists \varphi_i : A_i \subset \mathbb{R}^n \to Z$ Lipschitz

A_i Borel, $\varphi_i(A_i)$ mut. disj.

$\exists \theta_i \in L^1(A_i, \mathbb{N} \cup \{0\})$ $T = \sum_{i=1}^{\infty} \varphi_{i\#} [[\theta_i]]$.

Mass $\mathbb{M}(T) = \sum (\mathbb{M}(\varphi_{i\#} [[\theta_i]]))$.

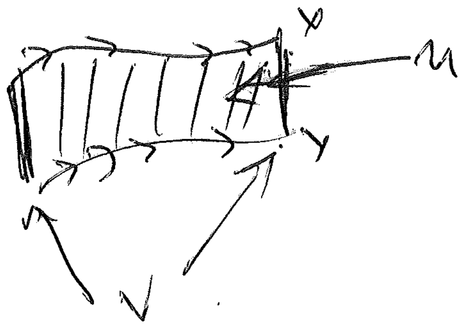
Def^{n} (Sormani - Wenger) (X, d, T) n-int-current sp.

T int current that satisfies

(I) ∂T current (II) set $T = X$

where set $T = \{x \in \bar{X} : \liminf_{r \to 0} \frac{\|T\|(B(x,r))}{\omega_n r^n} > 0\}$.

Ex. (Intrinsic flat dist). (X, d_X, T_X), (Y, d_Y, T_Y).



$d_F(T_X, T_Y) = \inf \{ \mathbb{M}(U) + \mathbb{M}(V) \}$.

$\partial U + V = T_X - T_Y$.

Th^{m} (Nakamichi, P). $(X_i, d_i) \in \text{Alex}^n(k, D)$ s.t.

$\exists T_i$ current with weights $= 1$. (ie $\theta_i = 1$).

s.t. (X_i, d_i, T_i) integral current spaces with $\partial T_i = 0$.

Then either (X_i, d_i, T_i) GH collapses or $\exists$ subsequence for which $GH = IF$. limits.