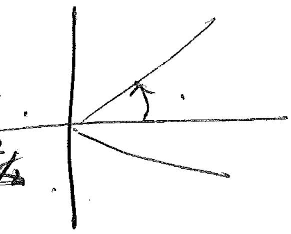
Andrea Carbonaro – H^∞ F.C. on L^p for contraction semigroups 16/09/2016.

$(T_t)_{t\geq 0}$ C_0 semigroup on $L^p(\mu)$.

$\|T(t)\| \leq 1$.

$-A_p$ generator, spec$(A_2) \subset S_\omega$, $\omega < \pi/2$.

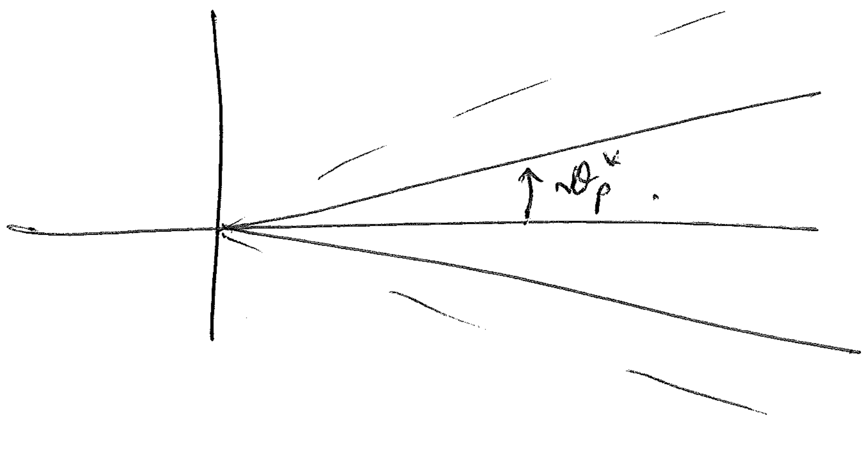


Lumer – Phillips : $\|T(z)\|_2 \leq 1$.

Complex interpolation. $\forall p \in (1,\infty)$ $\exists\, \vartheta_p > 0$ s.t.

$\|T(z)\|_p \leq 1$. $\forall z \in \overline{S}_{\vartheta_p}$ (ϑ_p lower angle.)

A_p sectorial with sectorial angle $\omega(A_p) \leq \theta_p^*$.



$\omega_H(A_p) = \inf\{\theta \in (\omega(A_p), \pi) : A_p$ has H^∞ F.C.$\}$.

→ ~~∀~~ All semigroups of analytic contractions:

- $w_H(A_p) \le \pi/2$. Fendler's dilation and C-W.
- $w_H(A_2) = w_H(A_2)$.

→ Complex interpolation due to Cowling:

$$w_H(A_p) < \pi/2.$$

Q: ① $p \in (1, \infty)$, $p \neq 2$, is $w_H(A_p) = w(A_p)$?
(1) Unknown, False on Reflexive B-spaces.

② Sharp lower bound for θ_p?

③ $w_H(A_p) \le \theta_p^*$?

$\theta_p = \pi/2 - \theta_p^*$, contractivity angle

Generators of symmetric contractions, answer ② & ③

$$\varphi_p^* = \arctan \frac{|p-2|}{2\sqrt{p-1}}. \qquad \varphi_p = \pi/2 - \varphi_p^*.$$

Kriegler 2011: $\|T(z)\|_p \le 1$. $\forall z \in \overline{S_{\varphi_p}}$

when T symmetric.

Contractive for all p and $A_2 = A_2^*$.

Thm — Generators of symmetric contraction semigroups.

$W_H(A_p) \le \varphi_p^*$, and $\theta > \varphi_p^*$, $m \in H^\infty(S_\theta)$

$$\| m(A_p) \| \lesssim \left(p^{9/4} \log p\right) \cdot \left(\theta - \varphi_p^*\right)^{-2}.$$

φ_p^* – sharp by Epperson's Thm.