Non-linear SPDEs, controlled distributions and renormalisation Part II

Hendrik Weber

Mathematics Institute
University of Warwick

Potsdam, 09.11.2013

Remember:

Regularity structure: Graded vector space $\mathbb{T}$ with a group of linear operators acting on it. List of objects in our local description.

Remember:

Regularity structure: Graded vector space $\mathbb{T}$ with a group of linear operators acting on it. List of objects in our local description.

Model: Maps an element in $\tau \in \mathbb{T}$ and points $x, y \in \mathbb{R}^d$ to a distribution $\Pi_x \tau$ and a $\Gamma_{x,y}$. Condition of decay around x for $\Pi_x \tau$ and bound on $\Gamma_{x,y}$ in terms of $|x - y|$.

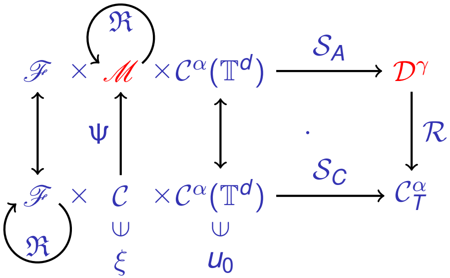
Remember:

Regularity structure: Graded vector space $\mathbb{T}$ with a group of linear operators acting on it. List of objects in our local description.

Model: Maps an element in $\tau \in \mathbb{T}$ and points $x, y \in \mathbb{R}^d$ to a distribution $\Pi_x \tau$ and a $\Gamma_{x,y}$. Condition of decay around x for $\Pi_x \tau$ and bound on $\Gamma_{x,y}$ in terms of $|x - y|$.

Modelled distribution: $\mathcal{D}^\gamma$. Mapping $f \colon \mathbb{R}^d \to \mathbb{T}$. Bound $\|f(x) - \Gamma_{x,y} f(y)\|_m \lesssim |x - y|^{\gamma - m}$.

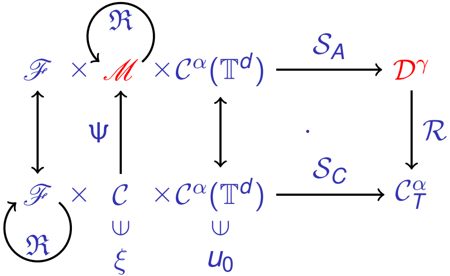
Where are we?



Missing:

- Need reconstruction operator $\mathcal{R}$.
- Need to define operations for modelled distributions:
  - Multiplication.
  - Integration against heat kernel.
- Need to build the right regularity structure for a given model.
- Need to check that renormalised models converge.

Where are we?



Missing:

- Need reconstruction operator $\mathcal{R}$.
- Need to define operations for modelled distributions:
  - Multiplication.
  - Integration against heat kernel.
- Need to build the right regularity structure for a given model.
- Need to check that renormalised models converge.

Reconstruction Theorem

$\mathbb{T} = (A, T, G) =$ regularity structure, $(\Pi, \Gamma) =$ model $\alpha = \min A$.

For every $\gamma > 0$, there exists a unique, continuous linear map $\mathcal{R} \colon \mathcal{D}^\gamma \to \mathcal{C}^\alpha$ such that (locally uniformly)

$$\big|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\big| \lesssim \delta^\gamma \|\Pi\| \, |\!|\!| f |\!|\!|_{\mathcal{D}_\gamma} \; .$$

$\mathcal{S}_x^\delta \eta =$ scaled testfunction.

Reconstruction Theorem

$\mathbb{T} = (A, T, G) =$ regularity structure, $(\Pi, \Gamma) =$ model $\alpha = \min A$.

For every $\gamma > 0$, there exists a unique, continuous linear map $\mathcal{R} \colon \mathcal{D}^\gamma \to \mathcal{C}^\alpha$ such that (locally uniformly)

$$\big|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\big| \lesssim \delta^\gamma \|\Pi\| \, |||f|||_{\mathcal{D}_\gamma} \; .$$

$\mathcal{S}_x^\delta \eta =$ scaled testfunction.

The condition $\gamma > 0$ corresponds to the condition $s + \alpha > 0$ in the multiplicative inequality for $u \in \mathcal{B}^\alpha_{\infty,\infty}$ and $v \in \mathcal{B}^s_{\infty,\infty}$.

Proof of uniqueness

$$\left|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\right| \lesssim \delta^\gamma \; .$$

- $f \in \mathcal{D}^\gamma$, ξ_1, ξ_2 two candidates for $\mathcal{R}f$. Set $\xi := \xi_1 - \xi_2$.

  Aim:

  $$\langle \xi, \psi \rangle = 0 \qquad \text{for all } \psi \text{ smooth.}$$

Proof of uniqueness

$$\left|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\right| \lesssim \delta^\gamma \;.$$

- $f \in \mathcal{D}^\gamma$, ξ_1, ξ_2 two candidates for $\mathcal{R}f$. Set $\xi := \xi_1 - \xi_2$.

  Aim:

  $$\langle \xi, \psi \rangle = 0 \qquad \text{for all } \psi \text{ smooth.}$$

- $\psi_\delta = \psi * \eta_\delta$. Then $\psi_\delta \to \psi$ and $\langle \xi, \psi_\delta \rangle \to \langle \xi, \psi_\delta \rangle$.

Proof of uniqueness

$$\big|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\big| \lesssim \delta^\gamma \;.$$

- $f \in \mathcal{D}^\gamma$, ξ_1, ξ_2 two candidates for $\mathcal{R}f$. Set $\xi := \xi_1 - \xi_2$.

  Aim:

  $$\langle \xi, \psi \rangle = 0 \qquad \text{for all } \psi \text{ smooth.}$$

- $\psi_\delta = \psi * \eta_\delta$. Then $\psi_\delta \to \psi$ and $\langle \xi, \psi_\delta \rangle \to \langle \xi, \psi_\delta \rangle$.
- $$\langle \xi, \psi_\delta \rangle = \int \xi * \eta_\delta(x)\, \psi(x) dx$$

Proof of uniqueness

$$\left|(\mathcal{R}f - \Pi_x f(x))(\mathcal{S}_x^\delta \eta)\right| \lesssim \delta^\gamma\ .$$

- $f \in \mathcal{D}^\gamma$, ξ_1, ξ_2 two candidates for $\mathcal{R}f$. Set $\xi := \xi_1 - \xi_2$.

  Aim:

  $$\langle \xi, \psi \rangle = 0 \qquad \text{for all } \psi \text{ smooth.}$$

- $\psi_\delta = \psi * \eta_\delta$. Then $\psi_\delta \to \psi$ and $\langle \xi, \psi_\delta \rangle \to \langle \xi, \psi_\delta \rangle$.

- $$\langle \xi, \psi_\delta \rangle = \int \underbrace{\xi * \eta_\delta(x)}_{\lesssim \delta^\gamma} \psi(x) dx \to 0.$$

Proof of existence

Aim: Given $f\colon \mathbb{R}^d \to \mathbb{T}$ construct $\mathcal{R}f \in \mathcal{D}'(\mathbb{R}^n)$.

Proof of existence

Aim: Given $f\colon \mathbb{R}^d \to \mathbb{T}$ construct $\mathcal{R}f \in \mathcal{D}'(\mathbb{R}^n)$.

- Approximate

$$\mathcal{R}_n f = \sum_{x \in 2^{-n}\mathbb{Z}} \langle \Pi_x f(x), \varphi_x^n \rangle \, \varphi_x^n.$$

Proof of existence

Aim: Given $f\colon \mathbb{R}^d \to \mathbb{T}$ construct $\mathcal{R}f \in \mathcal{D}'(\mathbb{R}^n)$.

- Approximate

$$\mathcal{R}_n f = \sum_{x \in 2^{-n}\mathbb{Z}} \langle \Pi_x f(x), \varphi_x^n \rangle \, \varphi_x^n.$$

- φ_x^n are Daubechies wavelets. These are rescalings of a regular function φ (e.g. $\mathcal{C}^r(\mathbb{R}^d)$ for r large) compactly supported and such that for there exits a_k

$$\varphi(x) = \sum_{k \in \mathbb{Z}^d} a_k \, \varphi(2x - k).$$

Proof of existence

Aim: Given $f\colon \mathbb{R}^d \to \mathbb{T}$ construct $\mathcal{R}f \in \mathcal{D}'(\mathbb{R}^n)$.

- Approximate

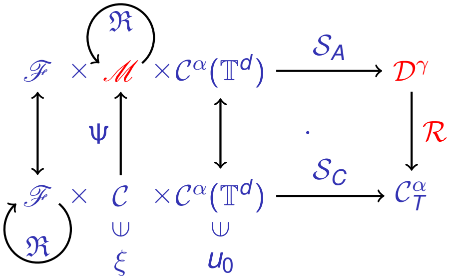
$$\mathcal{R}_n f = \sum_{x \in 2^{-n}\mathbb{Z}} \langle \Pi_x f(x), \varphi_x^n \rangle \, \varphi_x^n .$$

- φ_x^n are Daubechies wavelets. These are rescalings of a regular function φ (e.g. $\mathcal{C}^r(\mathbb{R}^d)$ for r large) compactly supported and such that for there exits a_k

$$\varphi(x) = \sum_{k \in \mathbb{Z}^d} a_k \, \varphi(2x - k).$$

- At every level n calculate $\mathcal{R}_{n+1} f - \mathcal{R}_n f$. Definitions are tailored to make this summable.

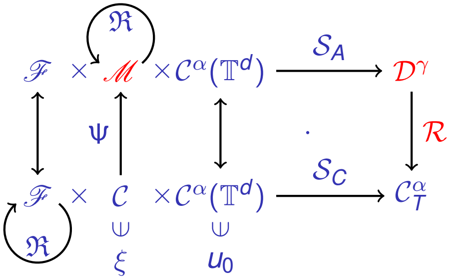
Where are we?



Missing:

- Need to define operations for modelled distributions:
  - Multiplication.
  - Integration against heat kernel.
- Need to build the right regularity structure for a given model.
- Need to check that renormalised models converge.

Where are we?



Missing:

- Need to define operations for modelled distributions:
  - Multiplication.
  - Integration against heat kernel.
- Need to build the right regularity structure for a given model.
- Need to check that renormalised models converge.

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta \Phi - \Phi^3 + \xi \, . \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta \Phi - \Phi^3 + \xi \;. \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_0 = 0.$$

$$\mathcal{W}_0 = \emptyset,$$

$$\mathcal{U}_0 = \emptyset,$$

$$\mathcal{F}_0 = \mathcal{W}_0 \cup \mathcal{U}_0 = \emptyset.$$

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta \Phi - \Phi^3 + \xi \,. \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_1 = K * \xi.$$

$$\mathcal{W}_1 = \{\Xi\},$$

$$\mathcal{U}_0 = \emptyset,$$

$$\mathcal{F}_0 = \mathcal{W}_0 \cup \mathcal{U}_0 = \emptyset.$$

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta \Phi - \Phi^3 + \xi \,. \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_1 = K * \xi.$$

$$\begin{aligned} \mathcal{W}_1 &= \{\Xi\}, \\ \mathcal{U}_1 &= \{X^k : k \in \mathbb{N}_0^3, \mathcal{I}(\Xi)\}, \\ \mathcal{F}_1 &= \mathcal{W}_1 \cup \mathcal{U}_1 = \{X^k : k \in \mathbb{N}_0^3, \Xi, \mathcal{I}(\Xi)\}. \end{aligned}$$

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta\Phi - \Phi^3 + \xi\ . \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_2 = K * ((K * \xi)^3 + \xi).$$

$$\mathcal{W}_2 = \{\Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3, X^k\mathcal{I}(\Xi), X^k\mathcal{I}(\Xi)^2\},$$

$$\mathcal{U}_1 = \{X^k, \mathcal{I}(\Xi)\},$$

$$\mathcal{F}_1 = \mathcal{W}_1 \cup \mathcal{U}_1 = \{X^k, \Xi, \mathcal{I}(\Xi)\}.$$

How is a regularity structure for an SPDE built?

$$\partial_t \Phi = \Delta\Phi - \Phi^3 + \xi\,. \qquad (\Phi^4_3)$$

Reinterpret as an integral equation (Duhamel's principle, mild solution, variation of constants)

$$\Phi = K * (\Phi^3 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_2 = K * ((K * \xi)^3 + \xi).$$

$$\mathcal{W}_2 = \{\Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3, X^k\mathcal{I}(\Xi), X^k\mathcal{I}(\Xi)^2\},$$

$$\mathcal{U}_2 = \{X^k, \mathcal{I}(\Xi)\} \cup \mathcal{I}(\mathcal{W}_2),$$

$$\mathcal{F}_2 = \mathcal{W}_2 \cup \mathcal{U}_2 = \ \ldots .$$

Φ^4_3 II

$$\partial_t \Phi = \Delta\Phi - \Phi^3 + \xi\,. \qquad (\Phi^4_3)$$

Final set of symbols:

$$\mathcal{F} := \cup_n \mathcal{F}_n = \{X^k, \Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3, X^k\mathcal{I}(\Xi), X^k\mathcal{I}(\Xi)^2, \ldots\}$$

Degree: For a symbol in $\mathcal{F}$ we define recursively

$$|\Xi| = -\frac{5}{2}\,, \quad |X^k| = |k|\,, \quad |\mathcal{I}(\tau)| = |\tau| + 2\,, \quad |\tau_1\tau_2| = |\tau_1|\,|\tau_2|\,.$$

Example: $|X^{k_1}\mathcal{I}(\mathcal{I}(\Xi)X^{k_2})| = |k_1| + |k_2| + 4 - \frac{5}{2}$.

Φ^4_3 II

$$\partial_t \Phi = \Delta\Phi - \Phi^3 + \xi\,. \qquad (\Phi^4_3)$$

Final set of symbols:

$$\mathcal{F} := \cup_n \mathcal{F}_n = \{X^k, \Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3, X^k\mathcal{I}(\Xi), X^k\mathcal{I}(\Xi)^2, \dots\}$$

Degree: For a symbol in $\mathcal{F}$ we define recursively

$$|\Xi| = -\frac{5}{2}\,, \quad |X^k| = |k|\,, \quad |\mathcal{I}(\tau)| = |\tau| + 2\,, \quad |\tau_1\tau_2| = |\tau_1|\,|\tau_2|\,.$$

Example: $|X^{k_1}\mathcal{I}(\mathcal{I}(\Xi)X^{k_2})| = |k_1| + |k_2| + 4 - \frac{5}{2}$.

Regularity structure:

$T_\alpha = \{$vector space generated by the symbols with weight $\alpha\,\}$.

Subcriticality ensures that T_α finite dimensional!

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_0 = 0.$$

$$\mathcal{W}_0 = \emptyset,$$

$$\mathcal{U}_0 = \emptyset,$$

$$\mathcal{F}_0 = \mathcal{W}_0 \cup \mathcal{U}_0 = \emptyset.$$

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_1 = K * \xi.$$

$$\mathcal{W}_1 = \{\Xi\},$$

$$\mathcal{U}_0 = \emptyset,$$

$$\mathcal{F}_0 = \mathcal{W}_0 \cup \mathcal{U}_0 = \emptyset.$$

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_1 = K * \xi.$$

$$\mathcal{W}_1 = \{\Xi\},$$
$$\mathcal{U}_1 = \{X^k, \mathcal{I}(\Xi)\},$$
$$\mathcal{F}_1 = \mathcal{W}_1 \cup \mathcal{U}_1 = \{X^k, \Xi, \mathcal{I}(\Xi)\}.$$

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_2 = K * ((\partial_x K * \xi)^2 + \xi).$$

$$\mathcal{W}_2 = \{X^k, \Xi, \mathcal{I}_1(\Xi), \mathcal{I}_1(\Xi)^2, \mathcal{I}_1(\Xi)X^k\},$$

$$\mathcal{U}_1 = \{X^k, \mathcal{I}(\Xi)\},$$

$$\mathcal{F}_1 = \mathcal{W}_1 \cup \mathcal{U}_1 = \{X^k, \Xi, \mathcal{I}(\Xi)\}.$$

How is a regularity structure for an SPDE built? KPZ

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Integral equation

$$\Phi = K * ((\partial_x h)^2 + \xi).$$

$K =$ heat kernel, $* =$ space-time convolution.

Start Picard iteration:

$$\Phi_2 = K * ((\partial_x K * \xi)^2 + \xi).$$

$$\mathcal{W}_2 = \{X^k, \Xi, \mathcal{I}_1(\Xi), \mathcal{I}_1(\Xi)^2, \mathcal{I}_1(\Xi)X^k\},$$

$$\mathcal{U}_2 = \{X^k, \mathcal{I}(\Xi)\} \cup \mathcal{I}(\mathcal{W}_2),$$

$$\mathcal{F}_2 = \mathcal{W}_2 \cup \mathcal{U}_2 = \ \ldots .$$

KPZ II

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Final set of symbols:

$$\mathcal{F} = \{X^k, \Xi, \mathcal{I}(\Xi), \mathcal{I}_1(\Xi), \mathcal{I}_1(\Xi)^2, \mathcal{I}_1(\Xi)X^k, \ldots\}$$

Degree: For a symbol in $\mathcal{F}$ we define recursively

$$|\Xi| = -\frac{3}{2} , \quad |X^k| = |k| , \quad |\mathcal{I}_1(\tau)| = |\tau| + 1 , \quad |\tau_1\tau_2| = |\tau_1|\,|\tau_2| .$$

Example: $|\mathcal{I}_1(\mathcal{I}(\Xi X^k)\mathcal{I}(\Xi))| = |k| + 2$.

KPZ II

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi, . \qquad \text{(KPZ)}$$

Final set of symbols:

$$\mathcal{F} = \{X^k, \Xi, \mathcal{I}(\Xi), \mathcal{I}_1(\Xi), \mathcal{I}_1(\Xi)^2, \mathcal{I}_1(\Xi)X^k, \dots\}$$

Degree: For a symbol in $\mathcal{F}$ we define recursively

$$|\Xi| = -\frac{3}{2}\,, \quad |X^k| = |k|\,, \quad |\mathcal{I}_1(\tau)| = |\tau| + 1\,, \quad |\tau_1\tau_2| = |\tau_1|\,|\tau_2|\,.$$

Example: $|\mathcal{I}_1(\mathcal{I}(\Xi X^k)\mathcal{I}(\Xi))| = |k| + 2$.

Regularity structure:

$T_\alpha = \{$vector space generated by the symbols with weight α $\}$.

Subcriticality ensures that T_α finite dimensional!

Which terms have to be constructed “by hand"?

Φ^4_3:

$$\mathcal{F} = \Big\{ \underbrace{X^k}_{|k|}, \underbrace{\Xi}_{-\frac{5}{2}}, \underbrace{\mathcal{I}(\Xi)}_{-\frac{1}{2}}, \underbrace{\mathcal{I}(\Xi)^2}_{-1}, \underbrace{\mathcal{I}(\Xi)^3}_{-\frac{3}{2}}, \underbrace{X_i\mathcal{I}(\Xi)}_{\frac{1}{2}}, \underbrace{X_i\mathcal{I}(\Xi)^2}_{0}, \underbrace{\mathcal{I}(\Xi)^2\mathcal{I}((\mathcal{I}(\Xi)^2)}_{0},$$
$$\underbrace{\mathcal{I}(\Xi)^2\mathcal{I}((\mathcal{I}(\Xi)^3)}_{-\frac{1}{2}}, \underbrace{\mathcal{I}(\Xi)\mathcal{I}((\mathcal{I}(\Xi)^3)}_{0} \Big\}$$

Terms of positive order can be dropped by reconstruction theorem.

Terms with divergent constants: $\mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3$, $\mathcal{I}(\Xi)^2\mathcal{I}(\mathcal{I}(\Xi)^2)$, $\mathcal{I}(\Xi)^2\mathcal{I}(\mathcal{I}(\Xi)^3)$.

Action of the group

The definition of the group action is slightly tricky.

Action of the group

The definition of the group action is slightly tricky.

- G acts on polynomials as before.

Action of the group

The definition of the group action is slightly tricky.

- G acts on polynomials as before.
- G acts trivially on $\Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3$ i.e. $\Gamma\Xi = \Xi$ etc.

Action of the group

The definition of the group action is slightly tricky.

- G acts on polynomials as before.
- G acts trivially on $\Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3$ i.e. $\Gamma\Xi = \Xi$ etc.

Terms with interaction of Ξ and X as well as $\mathcal{I}(\tau)$ with positive order become non-trivial to describe.

Action of the group

The definition of the group action is slightly tricky.

- G acts on polynomials as before.
- G acts trivially on $\Xi, \mathcal{I}(\Xi), \mathcal{I}(\Xi)^2, \mathcal{I}(\Xi)^3$ i.e. $\Gamma\Xi = \Xi$ etc.

Terms with interaction of Ξ and X as well as $\mathcal{I}(\tau)$ with positive order become non-trivial to describe.

Hopf-algebraic notation useful to capture the combinatorics.

Summary

- Very degenerate stochastic PDE arise as universal scaling limits in various models in statistical Physics.

Summary

- Very degenerate stochastic PDE arise as universal scaling limits in various models in statistical Physics.
- Due to irregularity of the white noise often not clear how to interpret non-linear terms.

Summary

- Very degenerate stochastic PDE arise as universal scaling limits in various models in statistical Physics.
- Due to irregularity of the white noise often not clear how to interpret non-linear terms.
- In subcritical (superrenormalisable) equations approximations converge if a finite number of “infinities" is removed.

Summary

- Very degenerate stochastic PDE arise as universal scaling limits in various models in statistical Physics.
- Due to irregularity of the white noise often not clear how to interpret non-linear terms.
- In subcritical (superrenormalisable) equations approximations converge if a finite number of “infinities" is removed.
- Regularity structures allow to separate efficiently the (probabilistic) analysis of the “perturbative" theory and the (deterministic) fixed point argument.