Renormalisation from Quantum Field Theory to Random and Dynamical Systems

November 6, 7, 8, 9 , 2013
(under the auspices of the University of Potsdam)

Foreword

The meeting aims at investigating the uses of renormalisation techniques inspired from physics beyond their original field of application, namely beyond quantum field theory. The need to renormalise arises in many a field such as in the theory of random PDEs and dynamical systems and in various disguises. Solving PDEs with singular (typically random) input or local dynamical systems via a linearisation procedure, involves taking limits of regularised expressions modified by the addition of diverging counterterms in order to ensure a convergence. These counterterms arise naturally through the action of a "renormalisation group", a concept borrowed from quantum field theory. The work of Dirk Kreimer and Alain Connes who gave an algebraic reformulation in the context of Hopf algebras, of the forest formula used by physicists, provides algebraic tools to organise the counterterms by means of a Birkhoff-Hopf factorisation. This algebraic approach to issues of an a priori purely analytic nature was the source of inspiration for further developments on the tree structure underlying the combinatorics of Feynman diagrams. The goal of this meeting is to provide an insight on the interaction between purely analytic renormalisation issues and the algebraic constructions used to approach them.

Acknowledgements

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The organisers: Sylvie Paycha, Michael Högele and Dominique Manchon.

ABSTRACTS

Colloquium talks

Nov. 6th University of Potsdam, Haus 9 Raum 1.02

Christian Brouder, Paris VI

S-matrix and renormalization

The S-matrix theory, proposed by Wheeler and Heisenberg, became a powerful approach to quantum field theory and renormalization in Lorentzian spacetime, after Stueckelberg and Bogoliubov realized how to take causality into account.

It turned out to be the correct framework to describe quantum field theory in curved spacetime.

Starting from a simple example, the renormalization of the S-matrix will be described. In particular, a large renormalization group will appear, which describes the way to change the interaction Hamiltonian of the system.

Hendrik Weber, University of Warwick

SPDEs, criticality, and renormalisation

Many models from statistical physics exhibit a behaviour called "phase transition". This means that the qualitative behaviour of a system changes drastically when one changes a model parameter just a little bit. One of the most prominent examples is the Ising model where spontaneous magnetisation is observed as soon as the temperature goes below the Curie temperature.

The behaviour of such a system near criticality often gives rise to interesting phenomena - long range correlations and non-Gaussian fluctuations can be observed. Sometimes such dynamics can be described by a non-linear white noise driven stochastic PDE such as the dynamic ϕ^4_d equations or the KPZ equation.

The analysis of these SPDEs is somewhat involved because they often do not make sense as they stand and infinities have to be subtracted in the right way. In this colloquium I will discuss several examples of such equations, discuss formally, why they arise as scaling limits of particle systems, and show in a simple example how this renormalisation procedure can be implemented.

Mini-courses
Nov. 7th, 8th, 9 th 2013, Kieszstr. 10, Potsdam

Loic Foissy, Reims
Algebraic and combinatorial aspects of quantum field theory

Many algebraic or combinatorial concepts are useful in Quantum Field theory:

1. Feynman graphs, which represent possible interactions between several elementary particles. They are endowed with combinatorial operations, such that insertion, extraction, or contraction.
2. These combinatorial operations are used to organize Feynman graphs inside a Hopf algebra. In this framework, Feynman rules are viewed as characters, and the renormalization process can entirely be seen as a Birkhoff decomposition on the character groups of these Hopf algebras. The Dyson-Schwinger equations of the Quantum Field Theory under study are de

   ned on these Hopf algebras, with the help of insertion operators.
3. These insertion operators defined on Feynman graphs are 1-cocycles for the Cartier- Quillen cohomology, and with the help of a universal property, this allows to replace Feynman graphs by rooted trees. We would like to explain how these structures allow for a description and a classification of all "physically meaningful" Dyson-Schwinger equations, that is to say of systems such that the unique solution generates a Hopf subalgebra of the Hopf algebra of Feynman graphs of the theory, and how this is related to the Faà di Bruno group of composition of formal series.

Massimiliano Gubinelli, Paris Dauphine
Paracontrolled distributions

We use the notion of para-product to introduce a class of random generalised functions and a calculus of non-linear operations on them which allows us to understand a few examples of singular random PDEs in a quite simple way. We will explain how to use these ideas to handle the KPZ equation, the stochastic quantization equation in 3 dimensions and a parabolic Anderson model in two dimensions. All these examples correspond to superrenormalisable theories from the point of view of naive power counting. We discuss the renormalisation procedures needed to render these equations well defined.

Frédéric Menous, Paris XI, Orsay
Renormalization and dynamical systems

The work of A. Connes and D. Kreimer in perturbative quantum field theory has made possible an algebraic interpretation of some renormalization schemes, as the Birkhoff decomposition of regularized characters, that is of elements of the group of algebra morphisms from a graded commutative Hopf algebra to the algebra of Laurent series. Many graded commutative Hopf algebras arise in this framework and the need for renormalization corresponds to the appearance of ill-defined characters. Surprisingly, the same objects (group of characters on Hopf algebras) arise in the study of analytic vector fields where changes of coordinates can be computed as elements either of the group of formal identity-tangent diffeomorphisms (namely characters of the Faà di Bruno Hopf algebra) or of some subgroup that corresponds to characters over a combinatorial Hopf algebra of trees or words. In the case of non-linearizable vector fields (resonant vector fields) the same problem of ill-defined characters appears. Such interactions should allow to enrich each domain with the ideas of the other and some steps have been made in that direction.

I will first give an introduction to dynamical systems and provide many elementary examples where some renormalization procedures give interesting results from the dynamical system point of view. I will then rephrase some problems on dynamical systems in the framework of graded complete Lie algebra, which are strongly linked to graded Hopf algebras. In this context, a specific renormalization procedure provides, from the dynamical point of view, normal forms associated with a given vector field as well as an analogue of the "beta function", for which a "locality" property arises.

Hendrik Weber, University of Warwick
Non-linear SPDEs, controlled distributions and renormalisation

Most of these two talks will be devoted to explain the theory of regularity structure, developed recently by M. Hairer. Classical notions of regularity for functions often capture how well a given function (or distribution) can be approximated by smooth functions. As a generalisation of this idea we will present the notion of controlled distribution. A distribution is "controlled" if it can be well approximated by - not necessarily smooth - functions in a prescribed set, the model. Based on this generalised concept of regularity, analytical notions like multiplication and convolution with a singular integral kernel are defined. This technique allows to construct solutions to a class of interesting non-linear stochastic PDEs, including the KPZ equation and the dynamic ϕ_3^4 model. These solutions are limits of suitable regularised solutions if one subtracts the right diverging counter-terms.

Talks

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Mohamed Belhaj Mohamed, University of Monastir, Tunisia

On renormalization groups and beta functions in a Hopf-algebraic context

We first give a review of the notions of locality, renormalization groups and beta functions for characters of a connected graded Hopf algebra. These notions, due to A. Connes and D. Kreimer, still make sense when the grading biderivation is replaced by another one. As an illustration of this remark, we'll detail the case of a semidirect product of two graded Hopf algebras, with rooted forests as a guiding example.

Viet Dang Nguyen, Paris VII

Renormalized products of distributions which fail to satisfy the Hörmander condition

Motivated by a problem in Quantum Field theory, we discuss the following analytical problem: how to multiply two distributions t_1 and t_2 on a manifold M whose wavefront sets $WF(t_1)$ and $WF(t_2)$ do not satisfy the Hörmander condition $WF(t_1) \cap -WF(t_2) = \emptyset$. We explain how provided the product makes sense (in the sense of Hörmander) on $M \setminus I$ for some submanifold I with dim $I <$ dim$M$, and t_1, t_2 satisfy some specific conditions related to their scaling behaviour, then we can construct a "renormalized" product $t_1 t_2$ over the manifold M whose analytical properties we will discuss.

Peter Imkeller, H.U. Berlin

A Fourier analytic approach to rough paths

In 1961, Ciesileski established a remarkable isomorphism of spaces of Hölder continuous functions and Banach spaces of real valued sequences. This isomorphism leads to wavelet decompositions of Gaussian processes giving access for instance to a precise study of their large deviations as shown by Baldi and Roynette. We will use Schauder representations for a pathwise approach of integration by means of Cisielki's isomorphism. It can be formulated in terms of dyadic martingales and Rademacher functions. In a more general and analytic setting, this pathwise approach of rough path analysis can be understood in terms of Paley- Littlewood decompositions of distributions, and Bony paraproducts in Besov spaces. This talk is based on work in progress with M. Gubinelli (U. Paris-Dauphine) and N. Perkowski (H.U. Berlin).

Erik Panzer, H.U. Berlin
Renormalization by kinematic subtraction and Hopf algebra

Apart from minimal subtraction, a common method of renormalization in physics is by subtraction at a fixed kinematical reference point. We will show how this method fits into the Hopf-algebraic setting and point out special properties of the renormalized Feynman rules obtained this way, working in a simplified setup first. In particular we relate the Hopf algebras of rooted trees and their universal property with the Hopf algebra of polynomials, leading us to the renormalization group. If time permits, we will compare the observed structures with the minimal subtraction scheme and explain how they look like in full quantum field theory.

Nicolas Perkowski, H.U. Berlin and Université Paris-Dauphine
Paracontrolled distributions and the parabolic Anderson model

The parabolic Anderson model describes the random motion of a particle in a random potential. On the lattice $\mathbb{Z}^d$, it is given as the solution to the linear heat equation, forced by a random potential. A natural scaling limit would be the continuous parabolic Anderson model, given as the solution to

$$\partial_t v(t,x) = \Delta v(t,x) + v(t,x)\xi(x)$$

where the spatial parameter x is in R^d,Δ is the Laplacian, and ξ is a spatial white noise. Unfortunately, this equation is ill-posed in dimensions $d > 1$, because then the product $v(t,x)\xi(x)$ cannot be defined using classical analytical results. I will show how to use the theory of paracontrolled distributions, which combines insights from the theory of controlled rough paths with paraproducts, to make sense of the continuous parabolic Anderson model in dimension $d = 2$, as well as some nonlinear version thereof. It turns out that we need to renormalize the equation by formally subtracting an infinite constant and solving for

$$\partial_t v(t;x) = \Delta v(t,x) + v(t,x)\xi(x) - \infty \cdot v(t,x)$$

This is joint work with Massimiliano Gubinelli and Peter Imkeller.

Jérémie Unterberger, Université de Nancy
Rough paths and renormalization

Stochastic differential equations (sde's) driven by white noise, i.e. by a noise which is decorrelated in time, arise in many areas of physics and engineering, and have been widely studied since the time of Itô. Solutions are known to be semimartingales, to which the standard tools of stochastic calculus – in particular, the competing Itô and Stratonovich stochastic integrations – apply. Considering now stochastic partial differential equations, or sde's driven by more irregular processes, one needs larger and larger excursions outside the realm of classical stochastic calculus in order to define solutions. Rough path theory, introduced by T. Lyons in 1998, has been developed as a general, geometric framework for defining integration along irregular paths, allowing e.g. (far from straightforward) extensions of such concepts as hypoellipticity or ergodicity from the case of classical diffusion equations to the case of sde's driven by irregular, colored noises. The general idea is that an integration procedure relies on a coherent choice of a finite number of iterated integrals of the noise, called *rough path*. Difficulties of a new type appear when the regularity index of the driving process goes below the threshold $\alpha = 1/4$; for such wildly oscillating processes, iterated integrals depend strongly on the highest frequency Fourier components, leading to divergences which have a natural reinterpretation as Feynman diagram ultra-violet divergences. Renormalization and constructive field theory, blended with probabilistic and combinatorial tools, give a satisfactory answer to these problems. We summarize here different approaches, with different scopes: general or specific to Gaussian processes (fractional Brownian motion with Hurst index $\leq 1/4$).